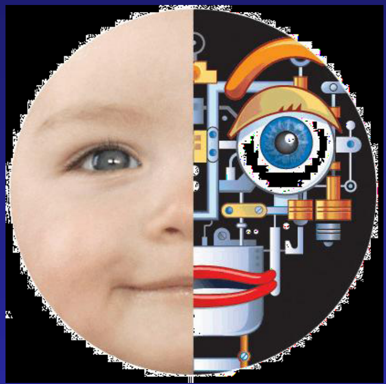
# Artificial Life



**Dr Richard Mitchell**

Soft, Hard and Wet (biological/chemical) approaches

Introductions, Theory and Applications




Cybernetics

# Aims of Module

**Aims:**

Swarm Intelligence and Artificial Life are two active areas of research in computational optimisation and modelling. This module aims to inspire students into exploring the creative potential of these fields as well as providing insight into the state-of-the-art.

So – will describe work in A-Life :

Overview + Soft, Hard, Wet

*Fundamentals & concepts, Progress & achievements*

And include latest research presentations –

theory and method ; advances and applications

# Assessment – 100% coursework

Presentation of academic paper : 30%

- Do in pairs (one group of 3, unless one wants to be on own)
- Find a recent relevant paper (journal/book chapter)
- Read paper and then develop 6 minute presentation on it
- Presentation to be given this Friday afternoon

Web Page : 70%

- For start of next term, develop web page on swarm intelligence and/or artificial life
- Must include novel applet (or video of applet) illustrating work
- Should be eye catching and interesting

Afternoons this week for finding paper, preparing presentation, as well as looking at notes, following up information

# History of A Life

Probably the first to actively study and write on related topics was John Von Neumann, mid 20th Century

- In “The General and Logical Theory of Automata” he proposed that living organisms are just machines.
- He also studied machine self replication, suggesting an organism must contain list of instructions on how to copy itseld
- Predating discovery of DNA (Crick, Watson, Franklin, Wilkins)

Also significant, Mathematical Games column in Scientific American, which publicised John Conway’s Cellular Automaton ideas (1960s)

The term ‘artificial life’ was coined by Chris Langton, late 1980s.

- He was also responsible for the first specific conference (on Synthesis and Simulation of Living Systems)

# ***What is Life ?***

*" What was life? No one knew. It was undoubtedly aware of itself, so soon as it was life; but it did not know what it was".*

Thomas Mann [1924]

*"Life is a dynamic state of matter organized by information".*

Manfred Eigen [1992]

*"Life is a complex system for information storage and processing".*

Minoru Kanehisa [2000]

The general condition that distinguishes organisms from inorganic objects and dead organisms, being manifested by growth through metabolism, a means of reproduction, and internal regulation in response to the environment.

Websters Dictionary (other defs also)

# What living things have in common

http://www.windows2universe.org/earth/Life/life1.html says biologists have determined that all living things share these:

Living things need to take in energy

Living things get rid of waste

Living things grow and develop

Living things respond to their environment

Living things reproduce and pass their traits onto their offspring

Over time, living things evolve (change slowly) in response to their environment

# *Also difficult: where did Life come from?*

**Geogenesis**:

Life started on Earth, in a relatively short period of time

Atomic synthesis of C, N, O elements complicated

Exact Conditions required to bootstrap life unknown

Not observed new life being created from elements

**Exogenesis**:

Life started on an equivalent of Earth

Life (or necessary components) travelled through space

Seeded life then flourished on Earth

**Panspermia** :

Life (and seeds of life) exists throughout the universe

Life could exist (and may already exist) elsewhere in the universe.

*Could A-Life help resolve the uncertainty?*

# Overview of ALife

**Promotion:**

Artificial Life, a field that seeks to increase the role of **synthesis** in the study of biological phenomena, has great potential, both for unlocking the secrets of life and for raising a host of disturbing issues-scientific and technical as well as philosophical and ethical.

Christopher G. Langton

**Academic:**

Artificial Life … investigates the scientific, engineering, philosophical, and social issues involved in our rapidly increasing technological ability to **synthesize** life-like behaviors from scratch in computers, machines, molecules, and other alternative media.

Artificial Life – Journal MIT press

**Synthesis:**

To make a synthesis of; to put together or combine into a complex whole; to make up by combination of parts or elements.

Oxford English Dictionary

# More from Chris Langton (1989)

AL views life as a property of the organization of matter, rather than a property of the matter which is so organized.

Whereas biology has largely concerned itself with the material basis of life, AL is concerned with the formal basis of life.

It starts at the bottom, viewing an organism as a large population of simple machines, and works upwards synthetically from there — constructing large aggregates of simple, rule-governed objects which interact with one another nonlinearly in the support of life-like, global dynamics.

The 'key' concept in AL is emergent behavior."

AL is concerned with tuning the behaviors of such low-level machines that the behavior that emerges at the global level is essentially the same as some behavior exhibited by a natural living system. [...] Artificial Life is concerned with generating lifelike behavior."

# *Alife Underview*

Alife is " Fact Free Science" John Maynard Smith, 1994

Instead of testing a hypothesis on observable data, Alife seeks to synthesise life like behaviour in agents.

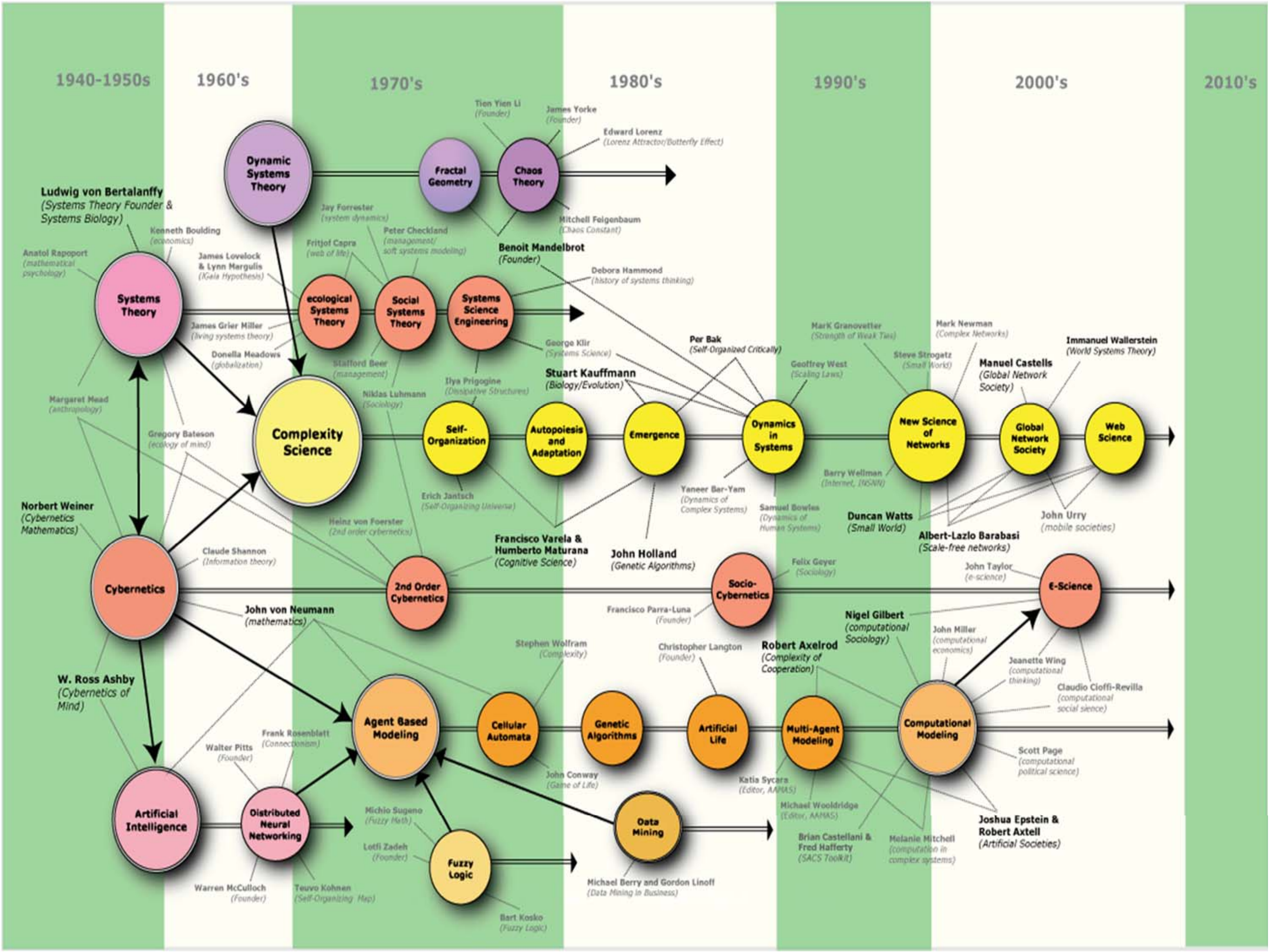
[Strong AL vs Weak AL debate as in with AI]

An agent has a set of assigned properties, components or abilities but not globally defined behaviours.

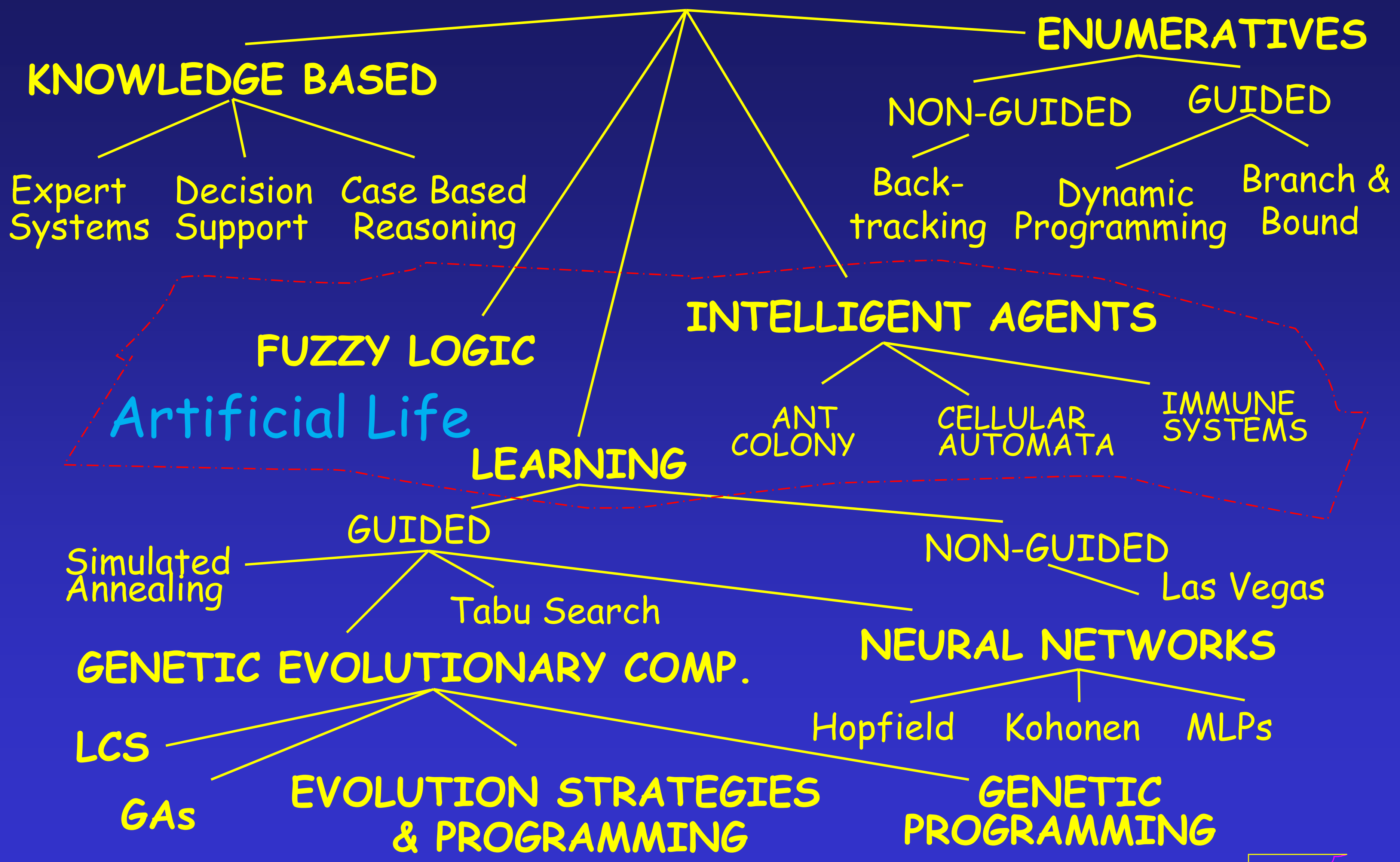
**Emergence** of global behaviours from local interactions is desired – Alife overlaps with Complex Systems

This 'bottom-up' approach with feedback & environmental interaction has similarities with Cybernetics

Next two slide shows where A Life fits with other disciplines.


1940-1950s
1960's
1970's
1980's
1990's
2000's
2010's
Dynamic Systems Theory
Fractal Geometry
Chaos Theory
Tien Yien Li (Founder)
James Yorke (Founder)
Edward Lorenz (Lorenz Attractor/Butterfly Effect)
Mitchell Feigenbaum (Chaos Constant)
Benoit Mandelbrot (Founder)
Ludwig von Bertalanffy (Systems Theory Founder & Systems Biology)
Kenneth Boulding (economics)
Anatol Rapoport (mathematical psychology)
James Lovelock & Lynn Margulis (Gaia Hypothesis)
Jay Forrester (system dynamics)
Fritjof Capra (web of life)
Peter Checkland (management/ soft systems modeling)
Debora Hammond (history of systems thinking)
Systems Theory
ecological Systems Theory
Social Systems Theory
Systems Science Engineering
James Grier Miller (living systems theory)
Donella Meadows (globalization)
Stafford Beer (management)
George Klir (Systems Science)
Per Bak (Self-Organized Criticality)
Mark Granovetter (Strength of Weak Ties)
Mark Newman (Complex Networks)
Immanuel Wallerstein (World Systems Theory)
Steve Strogatz (Small World)
Geoffrey West (Scaling Laws)
Manuel Castells (Global Network Society)
Stuart Kauffmann (Biology/Evolution)
Ilya Prigogine (Dissipative Structures)
Niklas Luhmann (Sociology)
Margaret Mead (anthropology)
Gregory Bateson (ecology of mind)
Complexity Science
Self-Organization
Autopoiesis and Adaptation
Emergence
Dynamics in Systems
New Science of Networks
Global Network Society
Web Science
Barry Wellman (Internet, INSNA)
Erich Jantsch (Self-Organizing Universe)
Yaneer Bar-Yam (Dynamics of Complex Systems)
Samuel Bowles (Dynamics of Human Systems)
Duncan Watts (Small World)
Albert-Lazlo Barabasi (Scale-free networks)
John Urry (mobile societies)
Norbert Weiner (Cybernetics Mathematics)
Heinz von Foerster (2nd order cybernetics)
Francisco Varela & Humberto Maturana (Cognitive Science)
John Holland (Genetic Algorithms)
Claude Shannon (Information theory)
Felix Geyer (Sociology)
John Taylor (e-science)
Cybernetics
2nd Order Cybernetics
Socio-Cybernetics
E-Science
John von Neumann (mathematics)
Francisco Parra-Luna (Founder)
Nigel Gilbert (computational Sociology)
John Miller (computational economics)
Stephen Wolfram (Complexity)
Christopher Langton (Founder)
Robert Axelrod (Complexity of Cooperation)
Jeanette Wing (computational thinking)
Claudio Cioffi-Revilla (computational social sience)
W. Ross Ashby (Cybernetics of Mind)
Agent Based Modeling
Cellular Automata
Genetic Algorithms
Artificial Life
Multi-Agent Modeling
Computational Modeling
Frank Rosenblatt (Connectionism)
Walter Pitts (Founder)
John Conway (Game of Life)
Katia Sycara (Editor, AAMAS)
Scott Page (computational political science)
Michio Sugeno (Fuzzy Math)
Michael Wooldridge (Editor, AAMAS)
Artificial Intelligence
Distributed Neural Networking
Data Mining
Lotfi Zadeh (Founder)
Fuzzy Logic
Brian Castellani & Fred Hafferty (SACS Toolkit)
Melanie Mitchell (computation in complex systems)
Joshua Epstein & Robert Axtell (Artificial Societies)
Warren McCulloch (Founder)
Teuvo Kohnen (Self-Organizing Map)
Michael Berry and Gordon Linoff (Data Mining in Business)
Bart Kosko (Fuzzy Logic)

# How A Life fits into AI



# Conference Topics - www.alifexi.org/cfp/

Synthesis and origin of life, self-organization, self-replication, artificial chemistries

Evolution and adaptation, evolutionary dynamics, evolutionary games, coevolution, major evolutionary transitions, ecosystems

Development, differentiation, regulation; generative representations

Synthetic biology

Self-organizing technology, self-computing/computational ecosystems

Unconventional and biologically inspired computing

Bio-inspired robots and embodied cognition, autonomous agents, evolutionary robotics

Collective behavior, communication, cooperation

Artificial consciousness; the relationship between life and mind

# Continued

Philosophical, ethical, and cultural implications

Mathematical and philosophical foundations of Alife

Evolution in the Brain; Artificial Consciousness: From Alife to Mind

Communication in Embodied Agents

Designing for Self;      Amorphous and Soft Robotics

Dynamical Systems Analysis

Trophic Interactions Between Digital Organisms

Autonomous Energy Management for Long Lived Robots

Models for Gaia Theory - including Daisyworld

The Environment and Evolution;      Hidden Epistemology

# *Artificial Life Already?*

**Soft**:

Cellular Automata,

Boids,

Evolutionary algorithms?

**Hard**:

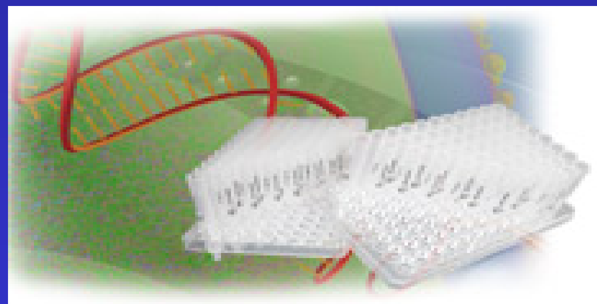
Self-replicating machines,

Self-building robots?

**Wet**:

Rat-brained robots,

DNA cartridges?



Not all criteria for life met.

Especially, adaptability: equilibrium is punctuated & truncated.

# *Soft A - Life*

*A Life Components*

Soft Hard Wet

Lets start with Software and Modelling

- flocking,
- cellular automata
- modelling daisyworld
- modelling 'real life'
- attractors and discrete models
- fractals and self-similarity

We start by looking at flocking

# *Flocking*

Alife about interacting systems … So flocking

**Collective motion**:

- Fish in schools, sheep in herds, birds in flocks, lobsters in lines

**Characteristics of animal aggregations**:

- Distinctive edges
- Freedom to move within own volume
- Coordinated movement

**Benefits of Flocking**

- Predator protection
- group foraging
- Social advantages – mating

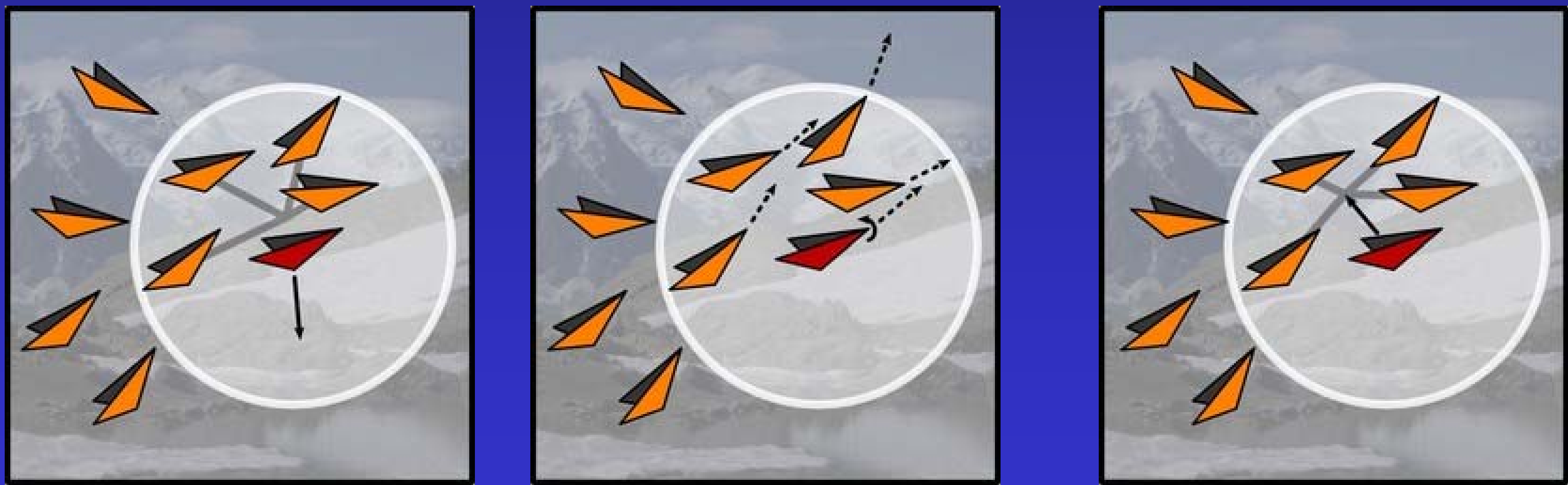
# *Craig Reynolds & "boids"*

http://cmol.nbi.dk/models/boids/boids.html    3 rules

**Separation**
*Steer to avoid crowding with local flock mates.*

**Alignment**
*Steer toward the average heading of local flock mates.*

**Cohesion**
*Steer to move toward average position of local flock mates.*

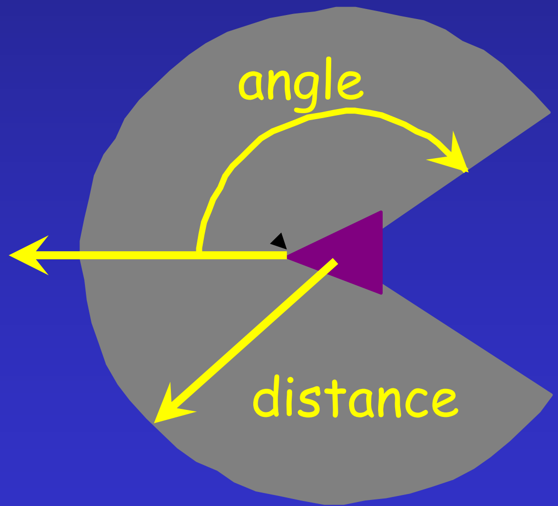


# Craig Reynolds & "boids"

Each boid has direct access to whole scene's geometric description

For Flocking react only to flockmates within its small neighborhood.

Neighbourhood = model of limited perception / region where flockmates influence steering




**distance**, measured from the center of the boid

**angle**, measured from boid's direction of flight

Flockmates outside local neighborhood ignored

http://www.red3d.com/cwr/boids/

http://dynamicnotions.blogspot.com/2008/12/flocking-boids-c.html

# *Critique of Flocking*

Reynolds' model is hypothetical, gives only appearance of flocking

Flocking is complex - inherent scaling problems

- Simple algorithm has asymptotic complexity of $O(n^2)$ each boid assesses each other boid to determine its neighbour
- Spatial data structure allows the boids to be kept sorted by their location reduces cost down to nearly $O(n)$

Lack of a quantitative model

- When is a flock a flock? Phase transition to become a flock ?
- When does flock change from cluster to V formation
- Heterogeneous vs homogeneous
- 300° vision cf. 360° vision

# *Langton : on Boids*

"Boids are *not birds; they are not even remotely like birds; they have no cohesive physical structure, but rather exist* as information structures — processes — within a computer.

But — and this is the critical 'but'— at the level of behaviors, *flocking Boids and flocking birds are two instances of the same phenomenon: flocking.*

The 'artificial' in Artificial Life refers to the component parts, not the emergent processes. If the component parts are implemented correctly, the processes they support are genuine — every bit as genuine as the natural processes they imitate.

Artificial Life will therefore be genuine life —it will simply be made of different stuff than the life that has evolved on Earth.

# *Cellular Automata*

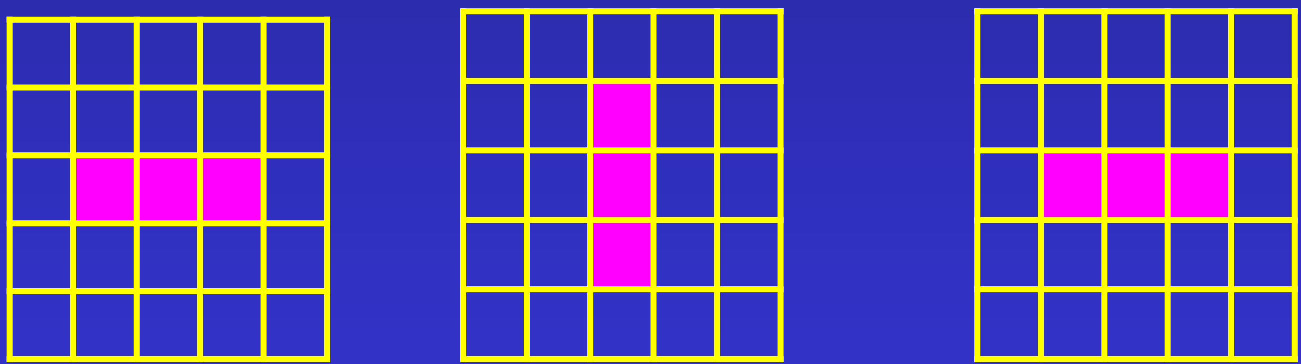
A regular grid of cells : each in finite states (often 0 or 1).

Commonly, 2D is used for the grid, higher dimensionality possible.

Time is discrete and the state of a cell at time t is a function of the states of a finite number of cells (neighborhood) at time t – 1

Every cell has the same rule for updating

Update based on neighbourhood (consider grid toroidal)



*t-2* *t-1* *t*

# Cellular Automata

John Von Neumann working at Los Alamos in the 1940s was interested in self-replicating robots

Stanislaw Ulam was working on crystal growth at the same time using a mathematical abstraction

Von Neumann created the first Cellular Automata (CA), but it was complex with 29 states per cell!

1970s John Conway greatly simplified CAs : Game of Life.

Practical uses have included studying crystal growth, casting of metals and biological patterns (e.g. coral)

'Fun' uses include pattern generation, screensavers and PhD studies

Theoretical uses have shown self replication, infinite growth and computational power.

# *Conway's Game of Life*

John Conway (Scientific American, 1970).

http://www.tech.org/~stuart/life/rules.html

Wanted a rule that for certain initial conditions would produce patterns that grow without limit, fade or get stable.

Have grid of cells which are occupied or not … have 8 neighbours

The rules for deriving a generation from the previous one are :

- Occupied cells with 0 or 1 occupied neighbours die of loneliness
- Occupied cells with 4..8 occupied neighbours die of overcrowding
- Otherwise occupied cells survive
- Unoccupied cells with 3 occupied neighbours come to life.

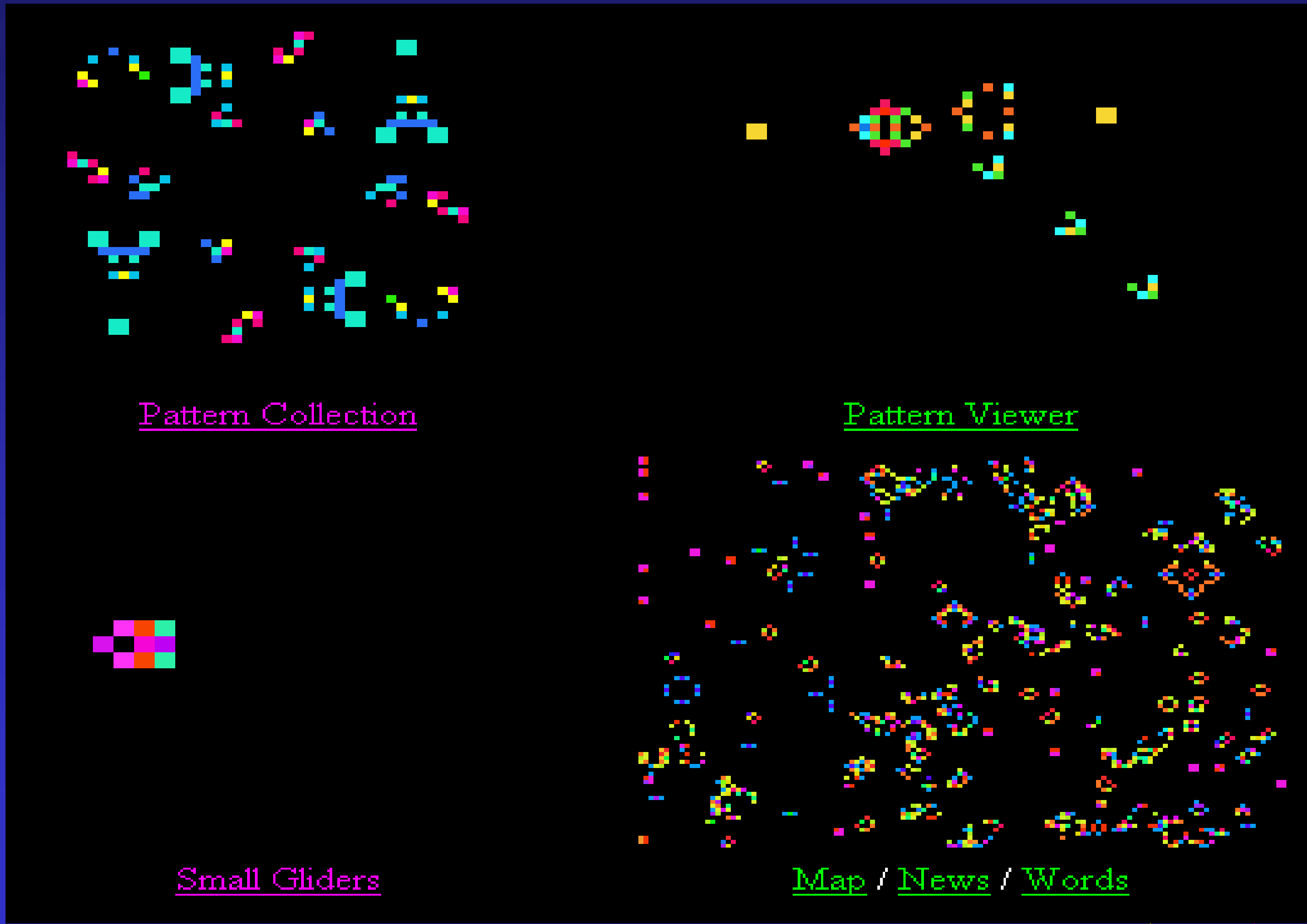
See code at http://blogs.msdn.com/calvin_hsia

# Gliders, guns and spaceships

Coloured Examples at http://www.collidoscope.com/cgolve/

Not just pretty patterns

Explores spontaneity, synchronicity and attractors



# Modern Cellular Automata Rule Notation

Modern CA software accepts multiple forms of rule specification.

Rules may be specified in either basic or canonical format.

Basic rule notation is based upon traditional "birth /survive".

"a" to "e" indicate the number of side neighbours in the rule.

"a" corresponds to zero side neighbours … "e" to four

Here are the meaningful combinations of total and side counts:

0a    1ab    2abc    3abcd

4abcde    5bcde    6cde    7de    8e

Here is full basic rule specification for Game Of Life:

3abcd / 2abc3abcd

Birth / Survive states

See http://www.collidoscope.com/modernca/

# Evolutionary Computation for ALife

Aim to find a solution to a particular problem:

Create a population of individuals to represent potential solutions

Evaluate the individuals

Introduce some selective pressure to promote better individuals (or eliminate lesser quality individuals)

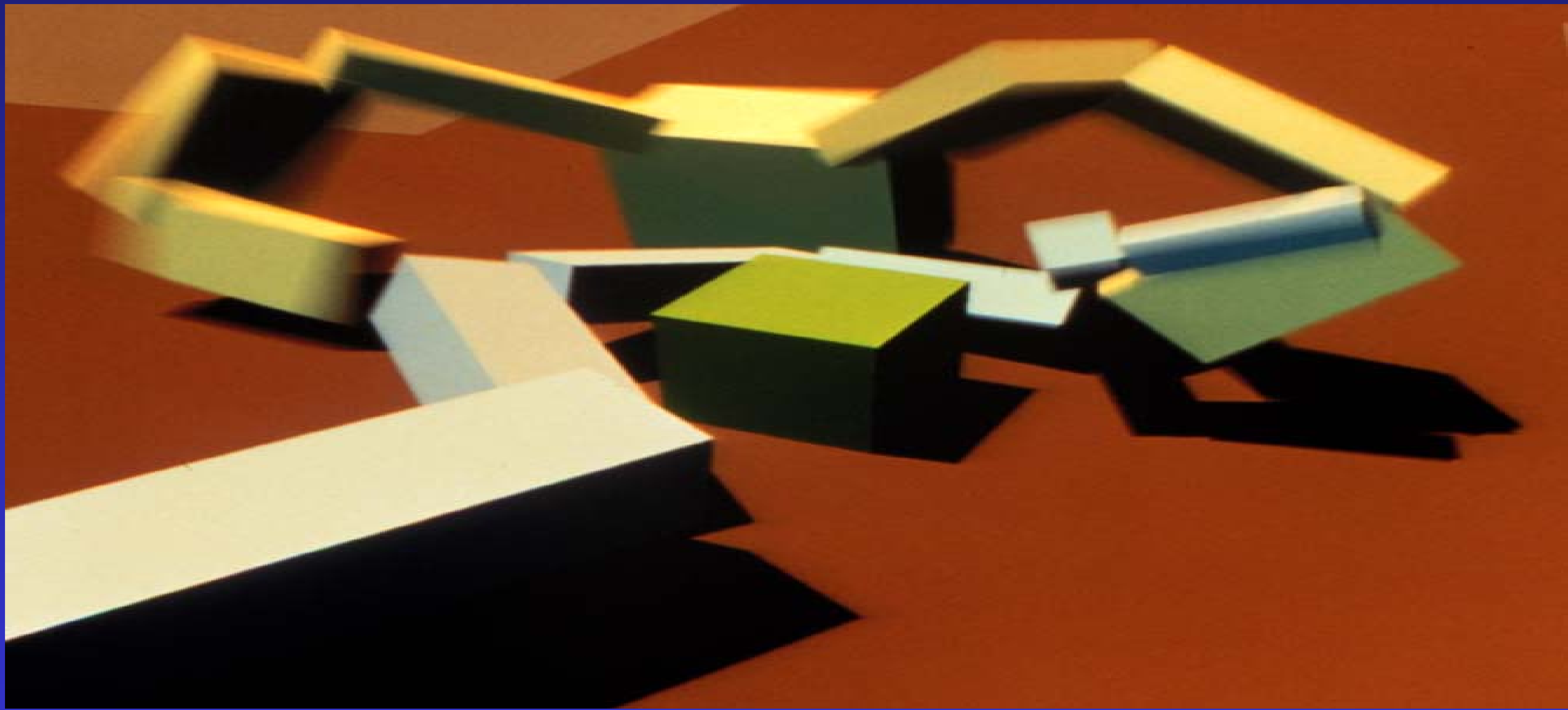
Apply some variation operators to generate new solutions

Repeat

# *Karl Sims*

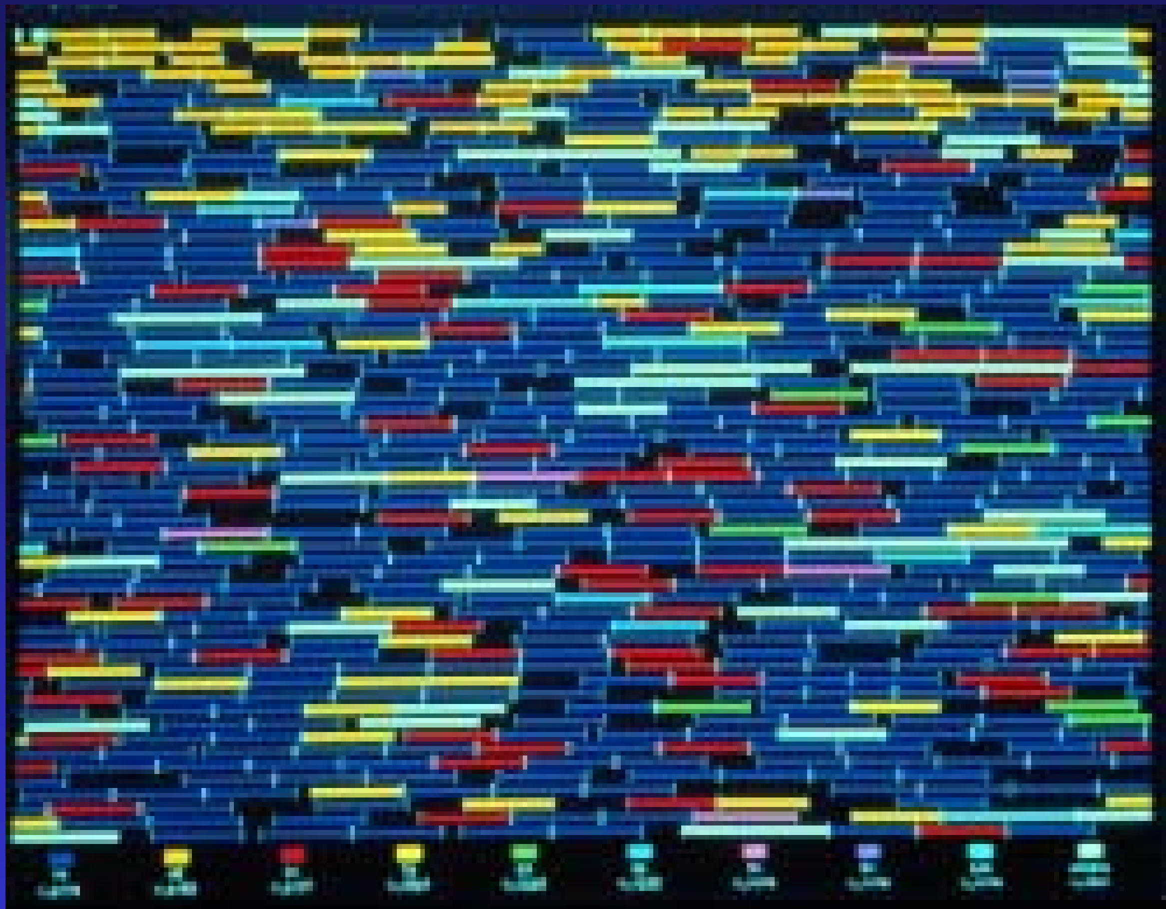
Evolution of physically realistic agents

Have populations comprising different components …



http://uk.youtube.com/watch?v=b1rHS3R0lIU

# Tierra - Tom Ray



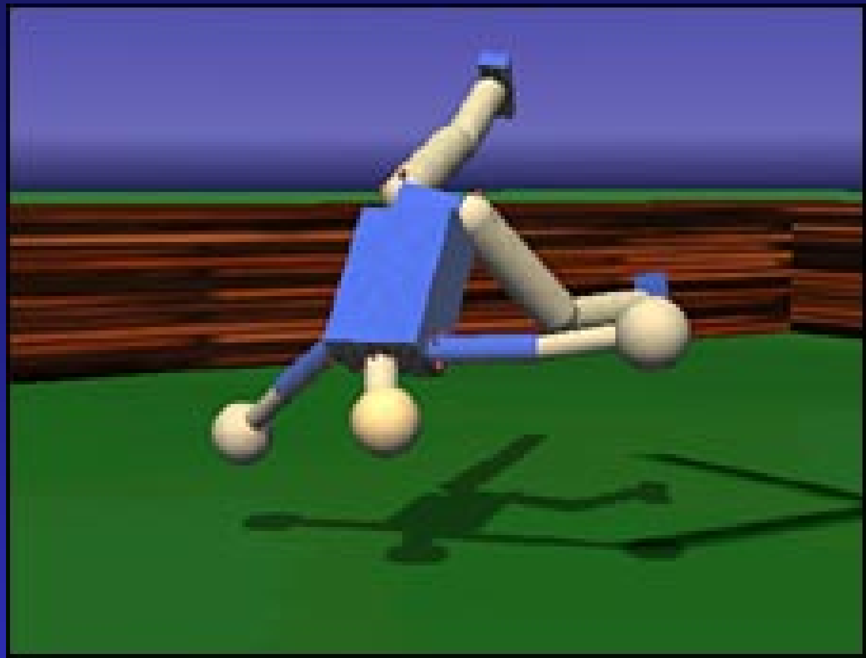
Evolution of memory based agents

Useful resources to view here

http://life.ou.edu/pubs/images/

# *Simulated Hardware*

Technically Soft
Alife
as in Karl Sims…



A NN effects motor commands and predicts next state

If agent encounters an unexpected obstacle it learns about itself / environment

Prof Ralf Der

http://news.bbc.co.uk/go/pr/fr/-/1/hi/technology/7544099.stm

# Summary

Have introduced Artificial Life

- Definitions
- Scope
- How relates to other disciplines
- Seen that it divides into Soft/Hard and Wet

We have started on Soft A-Life

- Flocking, Cellular Automata and some Evolutionary Computing
- Tomorrow look at more aspects …

Consider however the paper mentioned on the next slide … a good introduction to A-Life

# *Suggested Introductory Paper*

http://people.reed.edu/~mab/publications/papers/BedauTICS03.pdf

Artificial life: organization, adaptation and complexity from the bottom up by Mark A. Bedau

Artificial life attempts to understand the essential general properties of living systems by synthesizing life-like behavior in software, hardware and biochemicals. As many of the essential abstract properties of living systems (e.g. autonomous adaptive and intelligent behavior) are also studied by cognitive science, artificial life and cognitive science have an essential overlap. This review highlights the state of the art in artificial life with respect to dynamical hierarchies, molecular selforganization, evolutionary robotics, the evolution of complexity and language, and other practical applications. It also speculates about future connections between artificial life and cognitive science.

# 2 : More Soft ALife

Today Continue to look at Soft A Life

- Daisyworld
- Modelling 'real' life

Will build on this tomorrow with

- Attractors
- Discrete Models – and Self Similarity
- Fractals

# *Daisyworld*

Andrew J Watson and James E Lovelock; *Biological homeostasis of the global environment: the parable of Daisyworld*, Tellus (1983) 35B

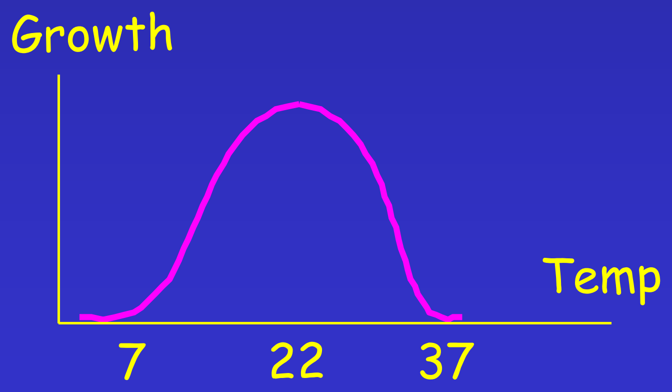
Lovelock's Imaginary world to demonstrate Gaia principle

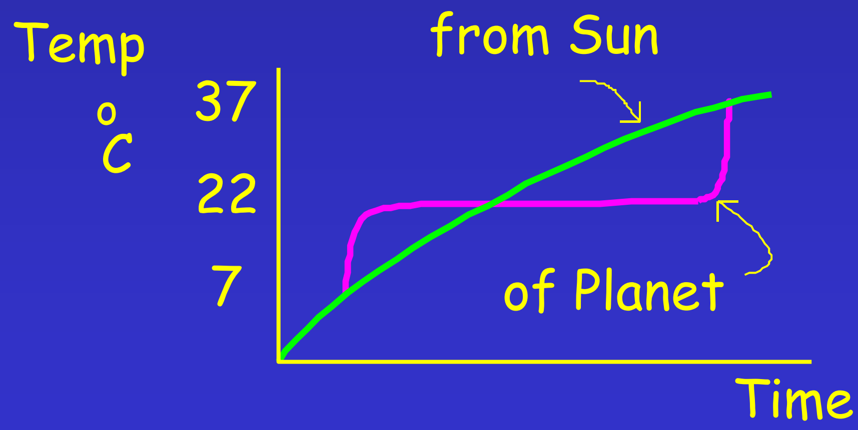
**Life & Earth work together to mutual advantage**

Grey Planet - black/white daisy seeds in soil

Daisies grow best at 22°C No grow if < 7°C or > 37°C

Daisyworld's Sun is heating up: What happens to Daisyworld?



# Modelling Life on Daisyworld

Model its temperature : model energy received, absorbed, emitted.

Energy received comes from the sun

Energy absorbed is affected by planet albedo

Planet albedo is affected by the areas of daisies

Areas affected by birth/death rates : affected by temperature

Energy Emitted (Stefan Boltzmann Law) $k * Temp^4$

Assume = Energy Absorbed = Energy Received - Energy Reflected

= Solar Luminosity * Solar Flux Const - Energy Received * Albedo

For World Temp, solve : StefansConst * $(WorldTemp + 273)^4$

= FluxConstant * Luminosity of Sun * (1 – Planet Albedo)

For any daisy species local temp is different re its albedo

$(Daisy\ Temp + 273)^4$ = AlbedoToTempConst *

(Planet Albedo - Daisy Albedo) + $(WorldTemp + 273)^4$

# *Albedos and Areas of Daisies*

Suppose have n species of Daisy

Let $D_i$ be area of Daisy Species i, $A_i$ its albedo, $T_i$ its temp

$D_0$ and $A_0$ can represent area and albedo of grey soil

$$\text{Planet Albedo} = \sum_{i=0}^{n} D_i * A_i$$

Areas of each daisy, found by solving differential equation

$$\frac{dD_i}{dt} = D_i * (\text{Uncolonised Fertile Soil} * \text{Birth rate} - \text{Death rate})$$

$$\text{Uncolonised Fertile Soil} = \text{Prop of Fertile Soil} - \sum_{i=1}^{n} D_i$$

$$\text{Birth Rate} = \text{Max}\,(0, 1 - 0.003265 * (22.5 - T_i)^2\ )$$

{Parabolic from $5^o$ - $40^o$, max at $22.5^o$}

# *Algorithm – at each solar time*

Initialise areas of daisies

Repeat

Calc Area Grey Soil, $D_0 = 1 - \sum_{i=1}^{n} D_i$

Calc Planet Albedo, $A = \sum_{i=0}^{n} A_i * D_i$

PlanetTemp, $PT = \sqrt[4]{\frac{\text{FluxConstant*luminosity*(1-Albedo)}}{\text{Stephan's Constant}}}$

For i = 1 to n Update $D_i$

Until all $D_i$'s reached steady value

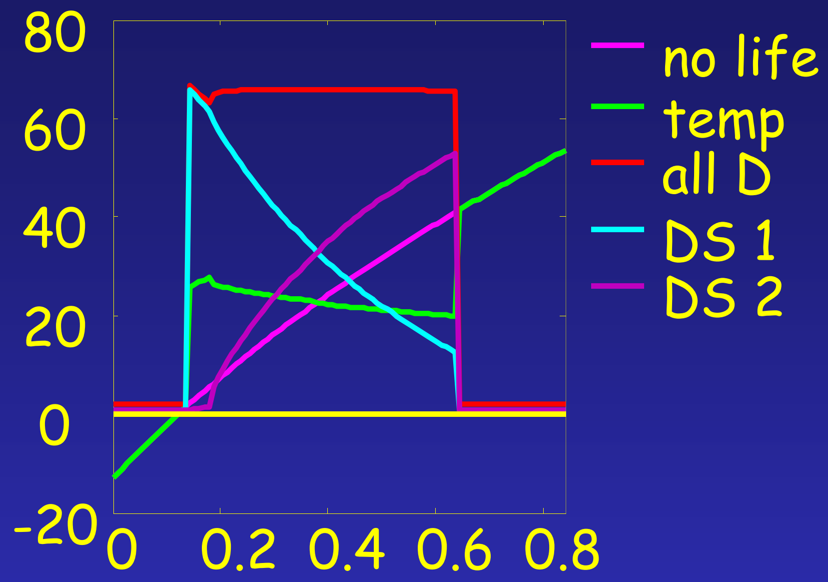
Update Di : Numerically integrate using

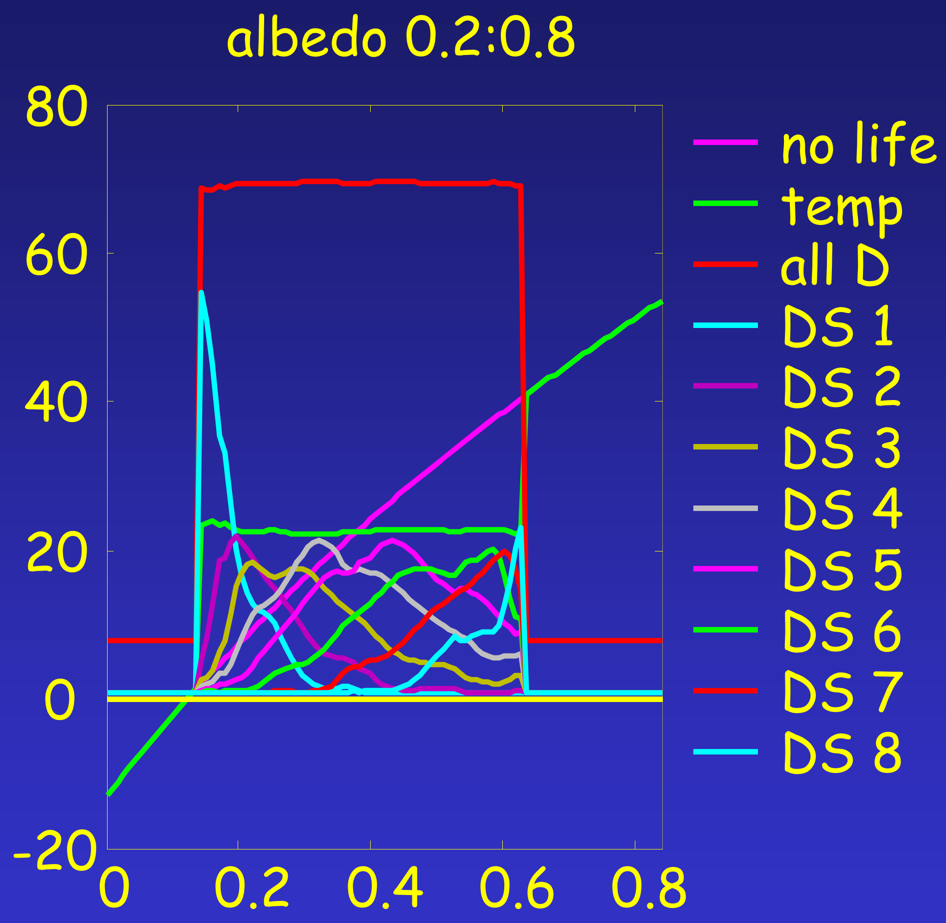
$T_i = \sqrt[4]{\text{AlbedoToTemp*(Albedo-}A_i) + PT^4} - 273$

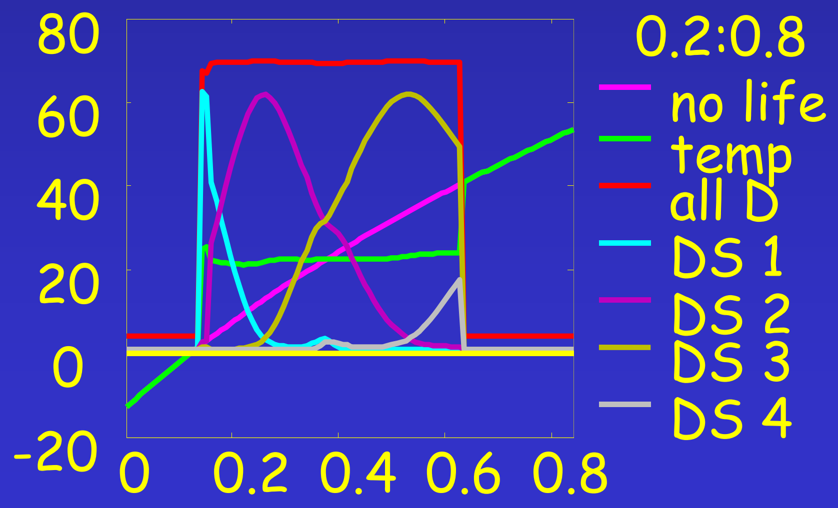
$\text{BirthRate} = 1.0 - 0.003265 * \text{Sqr}(22.5 - T_i)$

$\Delta = D_i * (\text{BirthRate} * \text{AreaFertileSoil} - \text{DeathRate})$

# Runs with 2, 4 or 8 species







Cybernetics

# Other Approaches / Extensions

Basic model : 'flat' earth – for sphere : divide into areas, each receiving different luminosity, and simulating each area.

Can daisies' albedo can evolve? See [Lenton is Lovelock's 'successor']

Lenton, T. M. 1998. Gaia and natural selection. Nature 394: 439-447

T.M.Lenton and J.E.Lovelock 2000. Daisyworld is Darwinian, Constraints on Adaptation are Important for Planetary Self-Regulation. J Theor Biol 206 109-114

At http://www.sussex.ac.uk/Users/jgd20/lisbon2007/

Computational Modelling of the Earth / Life System -

Includes – simulating Daisyworld using Cellular Automata

Also Homeostasis and Rein Control: From Daisyworld to Active Perception, by Inman Harvey, Proc ALife 9 2004

shows also how Rein control can be used for robotics

# Modelling Real Life

Consider population models and their analysis

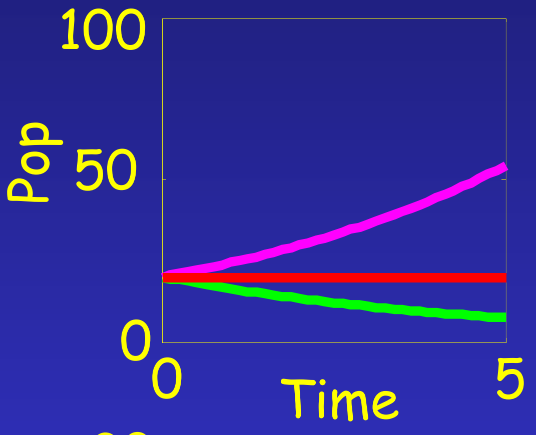
Inc. interacting species : predator-prey, mutualist, competitive

Starting with continuous models

Model by change of population P

$$\frac{dP}{dt} = (b - d) * P$$

b and d are birth and death rates

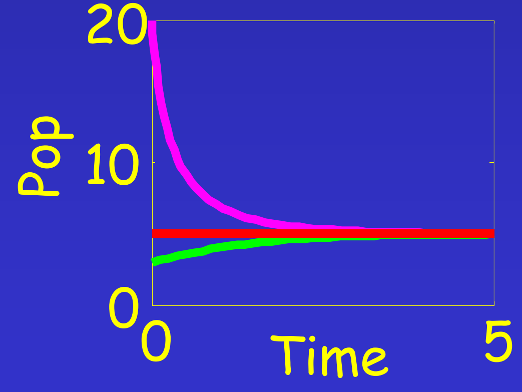



P constant, rises exponentially or decay to 0

If birth rate b - $b_2$ * P ; death rate d + $d_2$ * P.

$$\frac{dP}{dt} = (b - d \quad - \quad (b_2 + d_2)*P) \quad * \quad P$$

Pop stabilises at $\dfrac{b-d}{b_2 + d_2}$









Cybernetics

# *Classic Interacting Species*

Let F be number of foxes and R be number of rabbits.

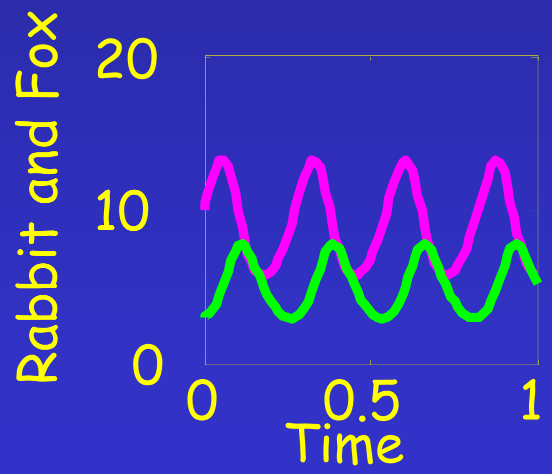
System model, as follows, where a, b, c, d are constants:

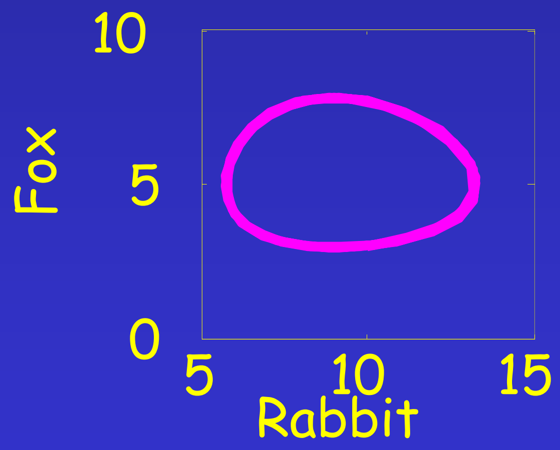
$$\frac{dR}{dt} = a*R-b*R*F \qquad \frac{dF}{dt} = c*R*F-d*F$$

Stable when
F = a/b and R = d/c

For a = 20, b = 4, c = 3 and d = 27: stable at 9,5;

Plot R and F v time but more useful Plot R v F (the phase plane plot)




Initial values
set size of 'egg'






Cybernetics

# Logistic Rabbit Model

If no Foxes, Rabbits increase exponentially – unrealistic, so

$$\frac{dR}{dt} = a*R - b*R*F - c*R^2 \qquad \frac{dF}{dt} = d*R*F-e*F$$

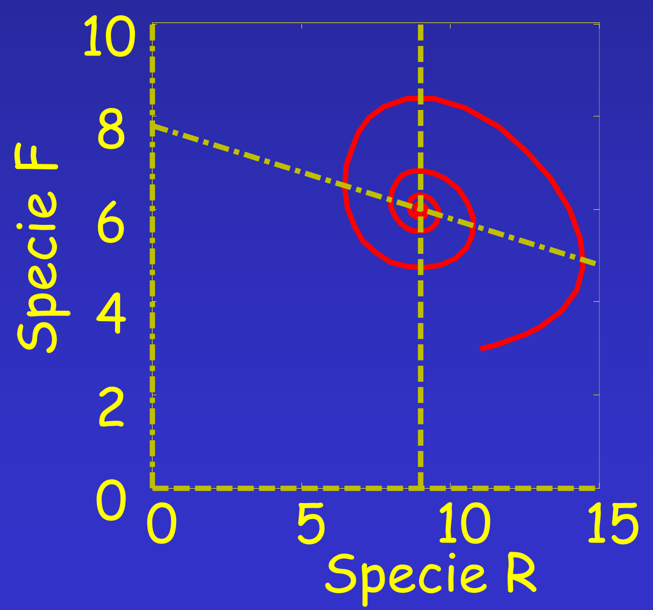
Suggests both populations stable at $R = \frac{e}{d}$ and $F = \frac{a-c\frac{e}{d}}{b}$

where lines a-bF-cR = 0

and dR-e=0 meet

$$\frac{dR}{dt} = R(39 - R- 5F)$$

$$\frac{dF}{dt} = F(-27 +3R)$$




With different constants, plot can go straight to equilibrium

# *Mutualist Interaction*

Interacting species which help each other

Also have commensalist (one helps other) systems.

Mutualists – some survive independently, sometimes reliant

| | |
|---|---|
| Eg Hippo and Bird | Clean teeth and food |
| Sea-anemone & damsel fish | Habitat + protection / food |
| Plants and Insects | Plant pollination insect food |
| Egret and Cattle | Egret eat insects on cattle. |

$$\frac{dx}{dt} = x\left(-13 - 2x^2 + 21y\right) \qquad \frac{dy}{dt} = y\left(-13 + 8x - 3y^2\right)$$

Analyse on phase plane, noting isoclines (loci where x and y const)

Equilibrium Points, where both x and y are constant

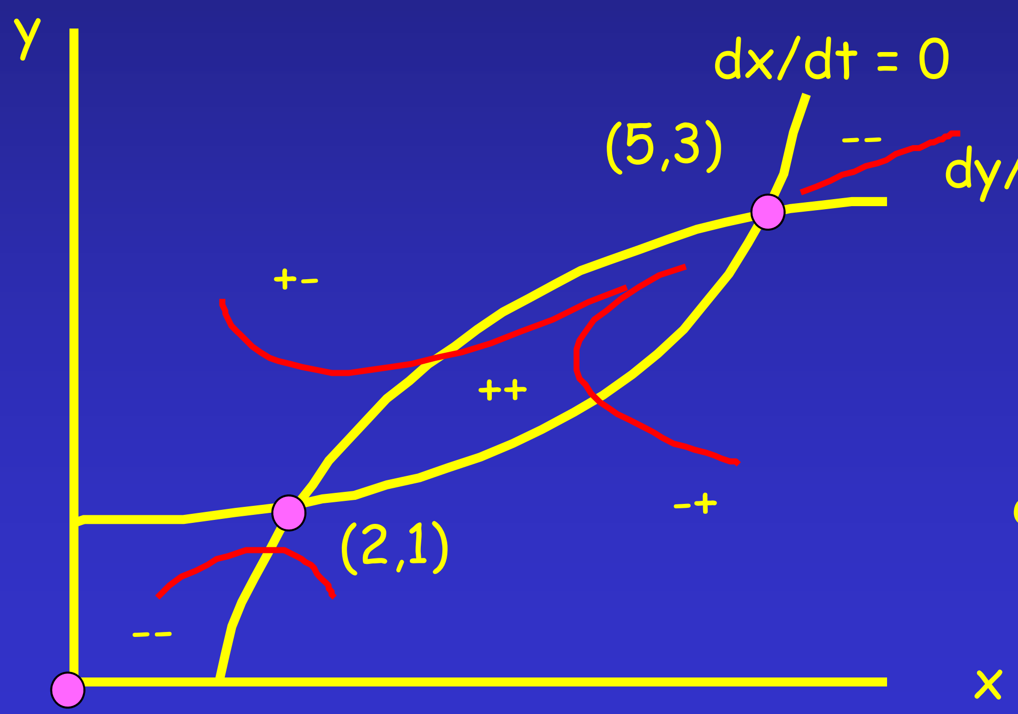
# Plot Zero Isoclines on Phase Plane

The iscolines for dx/dt are x = 0 and $-13 - 2x^2 + 21y = 0$

Those for dy/dt are y = 0 and $-12 + 8x - 3y^2 = 0$

Equilibium points: where a dx/dt isocline and a dy/dt isocline meet

Main iso's meet at 2,1 and 5,3; x = 0 and y = 0 meet at 0,0



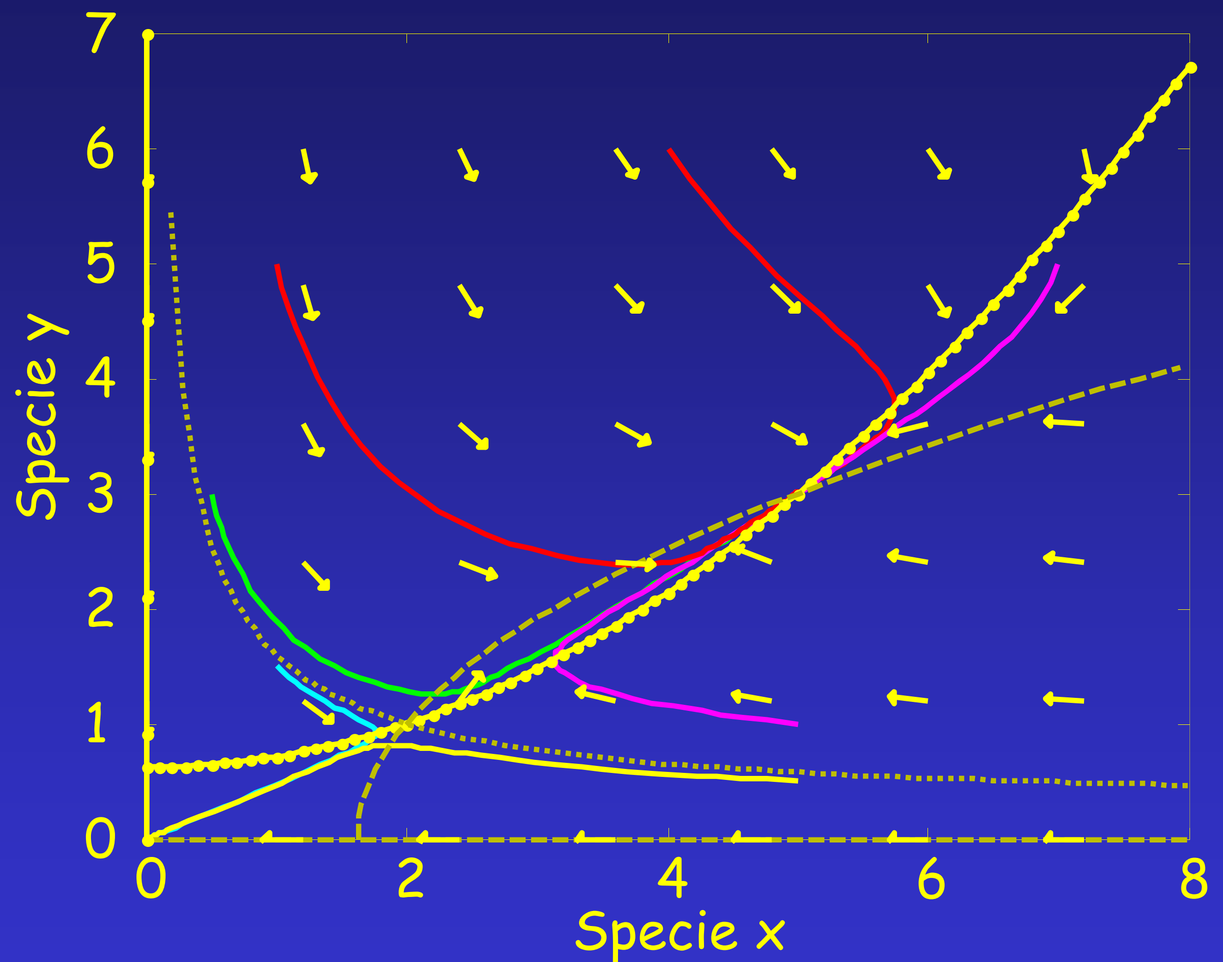

Show signs of x y

Argue how x,y change

Show two points stable

$$dx/dt = x(-13 - 2x^2 + 21y)$$
$$dy/dt = y(-13 + 8x - 3y^2)$$

# Go further : Phase Plane + Arrows




Arrows show dx/dt and dy/dt at intervals

Can be used to help sketch x and y values

See how x,y move from start posns

To 5,3 or 0,0

Not to 2,1

Note Separatrix (thru 2,1) : x,y → 5,3 if above this, else → 0,0







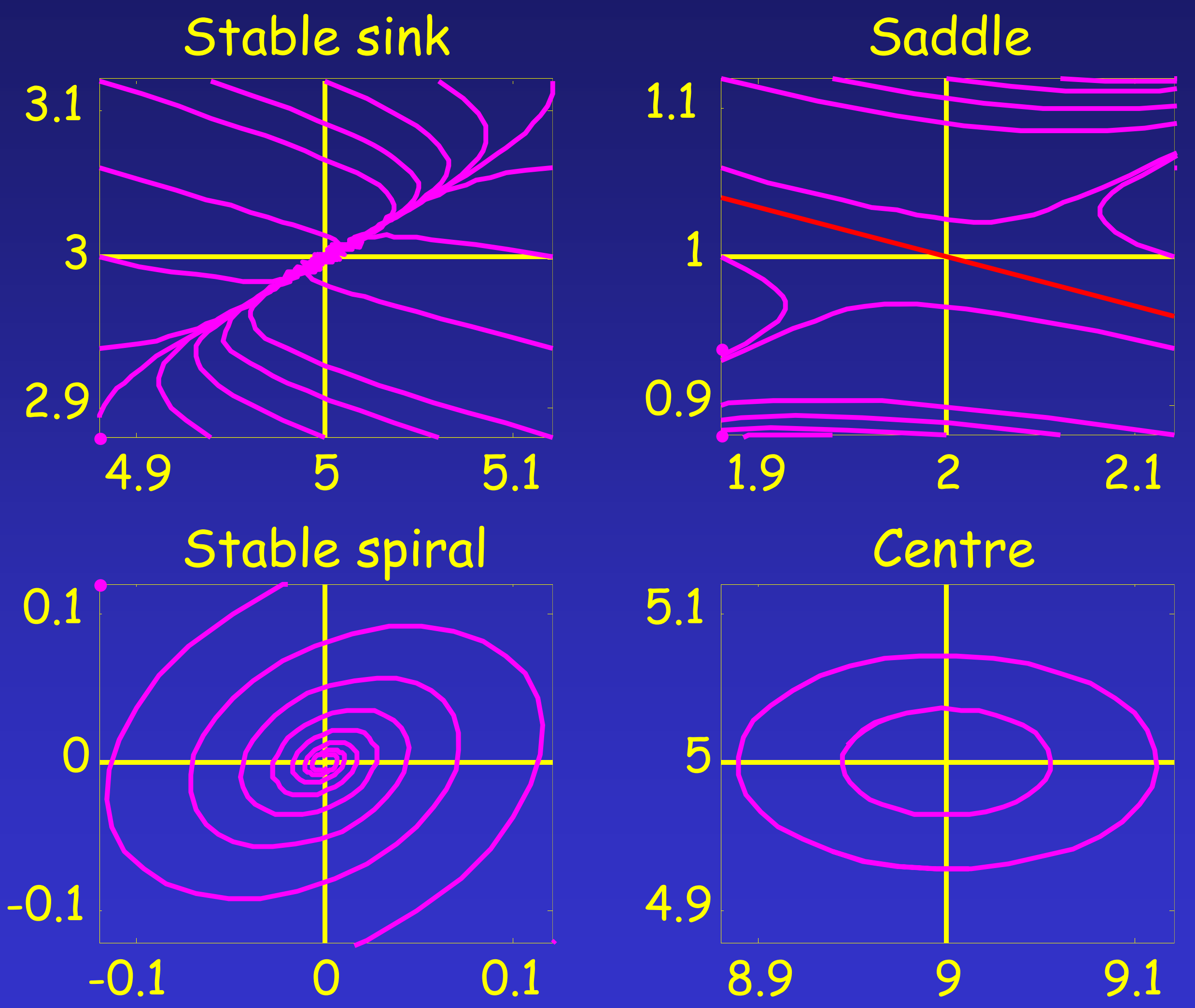
Cybernetics

# Types of Equilibrium Point

Where loci meet are equilibrium point – but different types exist

NB Also have unstable source and unstable spiral –

How can we categorise a point?



# Jacobean Matrix for Analysis

Define eq point as $(X_e,Y_e)$, here $F_1$ = dx/dt & $F_2$ = dy/dt = 0.

Model system as being linear around an equilibrium point

$$dx/dt = a_{11}X + a_{12}Y$$

$$dy/dt = a_{21}X + a_{22}Y$$

$$a_{11} = \frac{\partial F_1}{\partial x}\bigg|_{X_e,Y_e} \quad a_{12} = \frac{\partial F_1}{\partial y}\bigg|_{X_e,Y_e} \quad a_{21} = \frac{\partial F_2}{\partial x}\bigg|_{X_e,Y_e} \quad a_{22} = \frac{\partial F_2}{\partial y}\bigg|_{X_e,Y_e}$$

We then define two matrices **A** (the Jacobean) and **Z**

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \mathbf{Z} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

Then system of equations can be written as dZ/dt = **A Z**

# *Then Use Eigenvalues of Jacobean*

For a 2*2 matrix with eigenvalues $\lambda_1$ and $\lambda_2$:

- If $\lambda_1$ and $\lambda_2$ are both < 0, the equilibrium point is stable 'sink'
- If $\lambda_1$ and $\lambda_2$ are both > 0, the point is unstable 'source'
- If one < 0 and the other > 0, have a 'saddle' point
  - The eigenvector for $\lambda < 0$ is used for the separatrix
- If eigenvalues complex have spiral points –
  - stable (spiral in) if real($\lambda_1$ ) < 0, unstable otherwise
- If purely complex, have a 'centre'
  - For $\lambda^2 + b\lambda + c$ : stable sink if $b^2 \geq 4c$; else spiral in
  - For $\lambda^2 - b\lambda + c$ : source if $b^2 \geq 4c$; else spiral out
  - For $\lambda^2 + b\lambda - c$ : will be saddle point

# Analysis on the Example

$$\frac{dx}{dt} = F_1 = x\left(-13 - 2x^2 + 21y\right) \qquad \frac{dy}{dt} = F_2 = y\left(-13 + 8x - 3y^2\right)$$

$$\frac{\partial F_1}{\partial x} = (-13 - 2x^2 + 21y) + x(-4x) \qquad \frac{\partial F_1}{\partial y} = 21x$$

$$\frac{\partial F_2}{\partial y} = (-13 + 8x - 3y^2) - 6y^2 \qquad \frac{\partial F_2}{\partial x} = 8y$$

At 5,3, $\mathbf{A} = \begin{bmatrix} -100 & 105 \\ 24 & -54 \end{bmatrix}$ $\quad \lambda = -132.2178$ and $-21.7822$

So 5,3 is stable

At 2,1, $\mathbf{A} = \begin{bmatrix} -16 & 42 \\ 8 & -6 \end{bmatrix}$ $\quad \lambda = -30$ and $+8$

So 2,1 is a saddle point

Note Eigenvectors are $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 4 \end{bmatrix}$ for $\lambda = -30$ and 8

At 0,0, $\mathbf{A} = \begin{bmatrix} -13 & 0 \\ 0 & -13 \end{bmatrix}$ $\quad \lambda = -13$ and $-13$

So 0,0 is a stable point

# Separatrix

If start above this → 5,3; if start below → 0,0

The separatrix can be defined by dy/dx where

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Can't solve algebraically, so solve numerically

Run as ode from a known point on curve (ie saddle point)

One problem: at saddle point, dy/dx = 0/0;

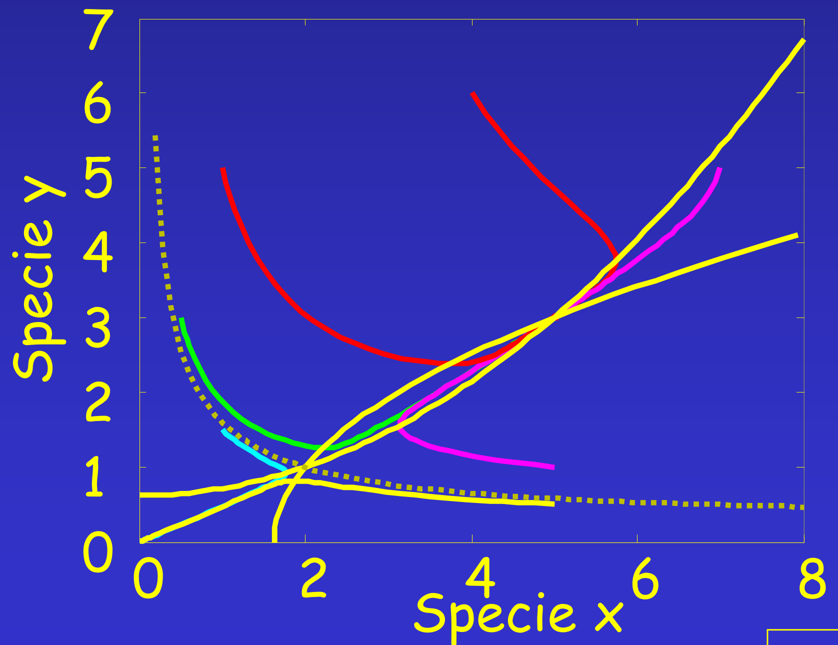
what is dy/dx?

*Answer, use its eigenvector*

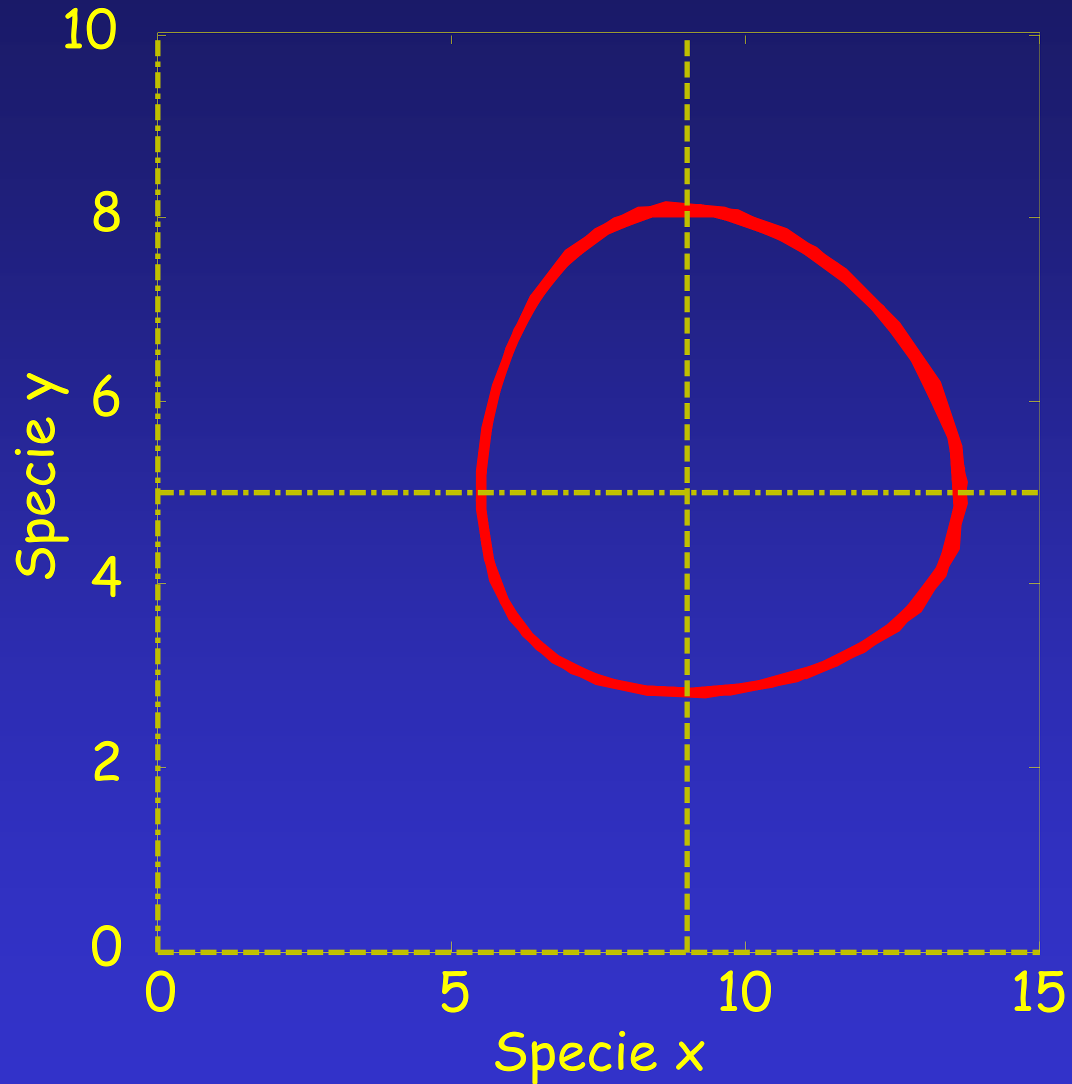
In example, at point 2,1

$$\Lambda = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\text{Slope} = -\frac{1}{3}$$



# Applies to Fox Rabbit




$$\frac{dR}{dt} = R(20 - 4F);$$

$$\frac{dF}{dt} = F(-27 + 3R)$$

At 9,5 $A = \begin{bmatrix} 0 & -36 \\ 15 & 0 \end{bmatrix}$

$\lambda = \pm\ 23.2379j$

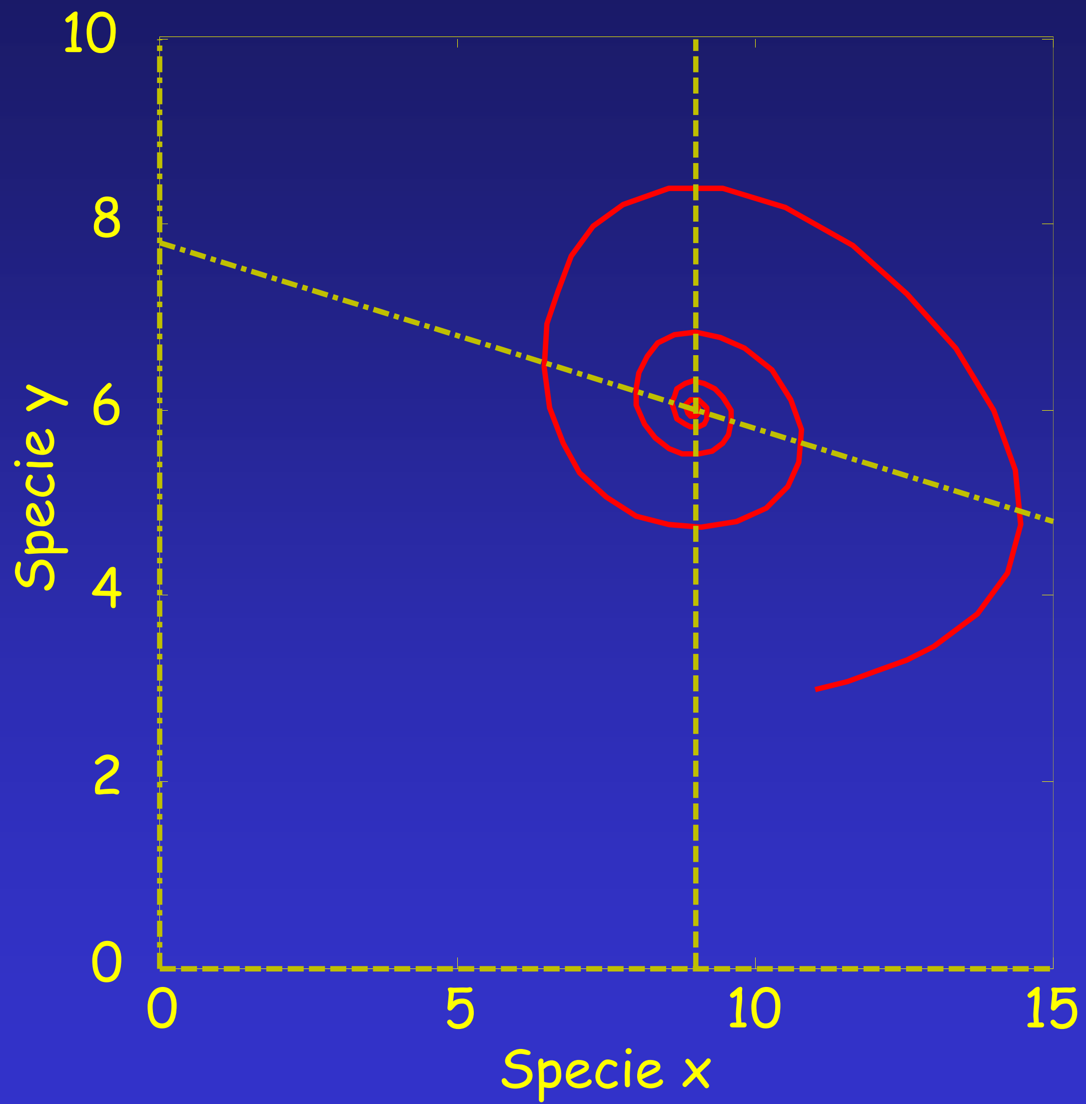
centre

At 0,0 $A = \begin{bmatrix} 20 & 0 \\ 0 & -27 \end{bmatrix}$

$\lambda$ = 20, -27

saddle

# Fox Rabbit (Logistic) Example




$$\frac{dR}{dt} = R(39 - R - 5F)$$

$$\frac{dF}{dt} = F(-27 + 3R)$$

At 9,6 $\quad A = \begin{bmatrix} -9 & -45 \\ 18 & 0 \end{bmatrix}$

$\lambda = -4.5000 \pm 28.1025j$

stable spiral

At 0,0 $\quad A = \begin{bmatrix} 39 & 0 \\ 0 & -27 \end{bmatrix}$

saddle



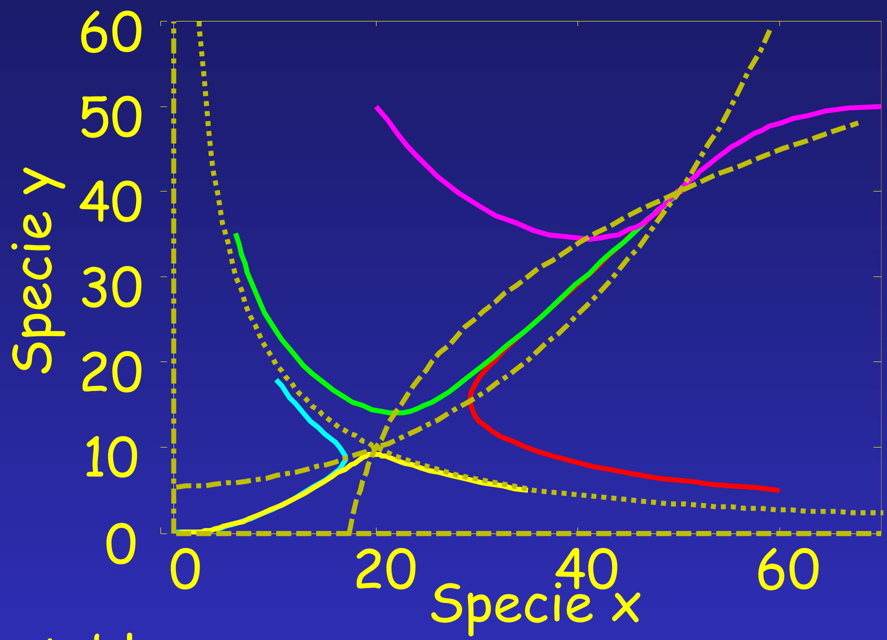

Cybernetics

# Models of Males and Females (M & F)

$$\frac{dM}{dt} = r_m FM - d_m M - k_m(M^3 + FM^2)$$

$$\frac{dF}{dt} = r_f FM - d_f F - k_f(F^3 + F^2 M)$$

e.g. $r_m = 1.3$, $d_m = 7$, $k_m = 0.01$,
$r_f = 1.1$, $d_f = 19$ and $k_f = 0.01$.




50,40: $\mathbf{A} = \begin{bmatrix} -70 & 40 \\ 28 & -52 \end{bmatrix}$ $\lambda = \begin{matrix} -95.7 \\ -26.3 \end{matrix}$ stable

20,10: $\mathbf{A} = \begin{bmatrix} -10 & 22 \\ 10 & -4 \end{bmatrix}$ $\lambda = \begin{matrix} -22.1 \\ 8.1 \end{matrix}$ saddle $\Lambda(-ve) = \begin{bmatrix} -0.88 \\ 0.48 \end{bmatrix}$

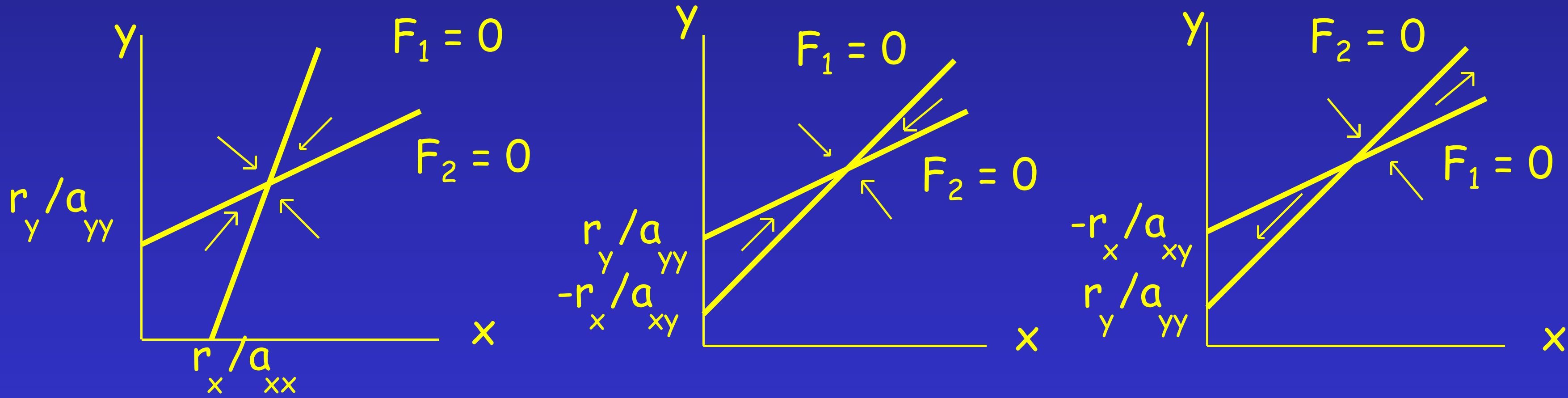
0,0: $\mathbf{A} = \begin{bmatrix} -7 & 0 \\ 0 & -19 \end{bmatrix}$ $\lambda = \begin{matrix} -7 \\ -19 \end{matrix}$ stable

# Lotka-Volterra Mutualism Models

$$F_1 = \frac{dx}{dt} = x\,(r_x - a_{xx}x + a_{xy}y) \qquad F_2 = \frac{dy}{dt} = y\,(r_y + a_{yx}x - a_{yy}y)$$

Isoclines are lines: eg when $x = 0$ or $r_x - a_{xx}x + a_{xy}y = 0$, etc.

Assume all 'a' parameters > 0. Consider 'main' equil. point




First 2 systems stable at main point, 3rd not. First ok with no mutualism; others obligate mutualists – can't exist on own

# *Advantage of Mutualism*

Note, with no mutualism (ie $a_{xy} = a_{yx} = 0$)

$$F_1 = \frac{dx}{dt} = x\,(r_x - a_{xx}x) \qquad \text{this is zero when } x = \frac{r_x}{a_{xx}}$$

$$F_2 = \frac{dy}{dt} = y\,(r_y - a_{yy}y) \qquad \text{this is zero when } y = \frac{r_y}{a_{yy}}$$

But, because of the positive feedback, due to mutualism, stable point is higher than these values.

Note, we will show, if opposite can be true in competitive systems

Next slide shows EQ point stable if gradient of $F_1$ isocline > $F_2$'s

If isoclines are curves, this gradient test also applies …

# *For Info : Confirm Gradient Test*

A matrix is $\begin{bmatrix} -a_{xx}X_e & a_{xy}X_e \\ a_{yx}Y_e & -a_{yy}Y_e \end{bmatrix}$

Char Eqn : $(-a_{xx}X_e - \lambda)(-a_{yy}Y_e - \lambda) - a_{xy}X_e a_{yx}Y_e = 0$

$$\lambda^2 + \lambda(a_{xx}X_e + a_{yy}Y_e) + a_{xx}X_e a_{yy}Y_e - a_{xy}X_e a_{yx}Y_e = 0$$

To be stable, need two negative eigenvalues, ie

$$a_{xx}X_e a_{yy}Y_e > a_{xy}X_e a_{yx}Y_e$$

ie $a_{xx}a_{yy} > a_{xy}a_{yx}$ or $\dfrac{a_{xx}}{a_{xy}} > \dfrac{a_{yx}}{a_{yy}}$

ie gradient of $F_1$ isocline > that of $F_2$ isocline

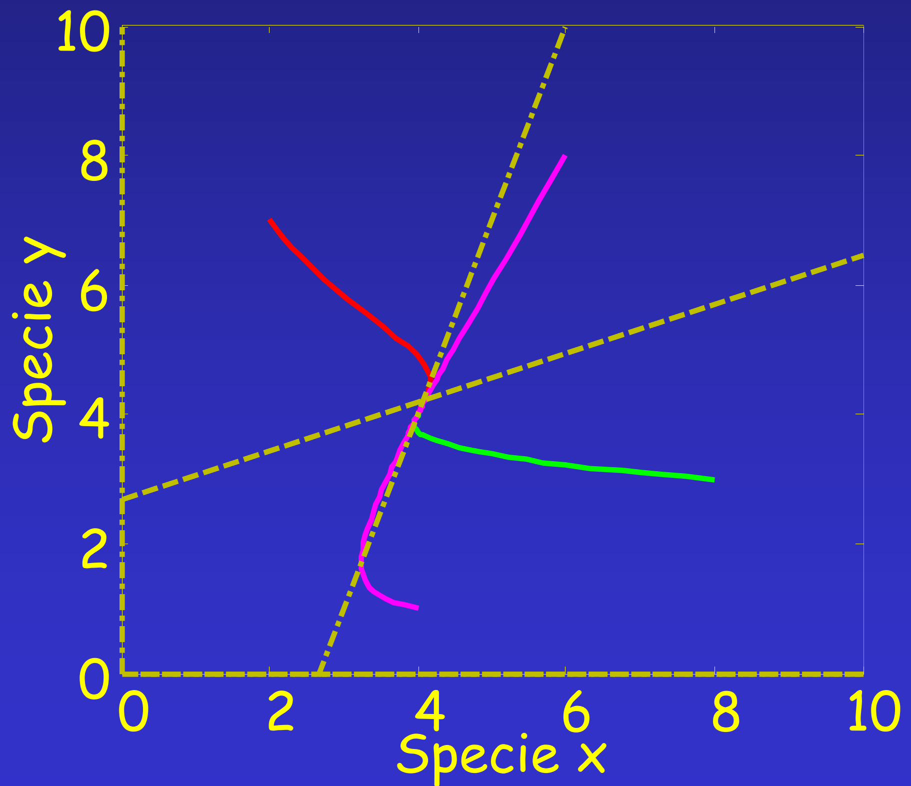



Cybernetics

# *Lotka-Volterra Example*

$$F_1 = \frac{dx}{dt} = x(-7x + 2y + 20)$$

$$F_2 = \frac{dy}{dt} = y(x - 3y + 8)$$




At 4,4: $A = \begin{bmatrix} -28 & 8 \\ 4 & -12 \end{bmatrix}$

$\lambda$ = -29.8 -10.2 stable

At $0, \frac{8}{3}$: $A = \begin{bmatrix} 25.33 & 0 \\ 2.667 & -8 \end{bmatrix}$

$\lambda$ = -8 25.333 saddle

At $\frac{20}{7}$,0: $A = \begin{bmatrix} -20 & 5.714 \\ 0 & 10.86 \end{bmatrix}$

$\lambda$ = -20 10.86 saddle

At 0,0: $A = \begin{bmatrix} 20 & 0 \\ 0 & 8 \end{bmatrix}$

$\lambda$ = 20 8 unstable

# Competitive Species

$$\frac{dx}{dt} = x(8 - 0.5x - 2y) \qquad \frac{dy}{dt} = y(10 - x - 2y)$$

$$\frac{\partial F_1}{\partial x} = 8\text{-}x\text{-}2y \qquad \frac{\partial F_1}{\partial y} = -2x \qquad \frac{\partial F_2}{\partial x} = \text{-}y \qquad \frac{\partial F_2}{\partial y} = 10\text{-}x\text{-}4y$$

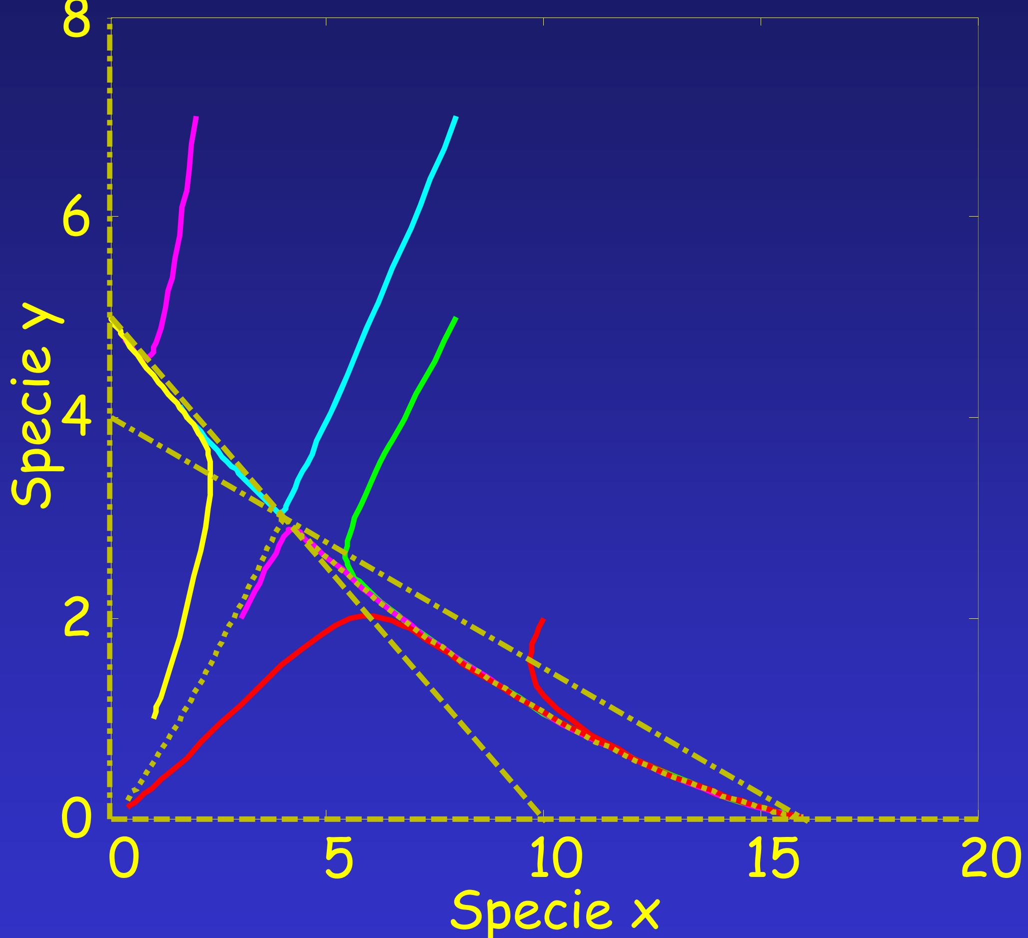
At 4,3, $\boldsymbol{A} = \begin{bmatrix} -2 & -8 \\ -3 & -6 \end{bmatrix}$ $\lambda$ = 1.29 and -9.29 saddle

At 16,0, $\boldsymbol{A} = \begin{bmatrix} -8 & -32 \\ 0 & -6 \end{bmatrix}$ $\lambda$ = -8, -6 stable

At 0,5, $\boldsymbol{A} = \begin{bmatrix} -2 & 0 \\ -5 & -10 \end{bmatrix}$ $\lambda$ = -10, -2 stable

At 0,0, $\boldsymbol{A} = \begin{bmatrix} 8 & 0 \\ 0 & 10 \end{bmatrix}$ $\lambda$ = 8,10 unstable

# *Result – only one species survives*




At 4,3 Saddle $A = \begin{bmatrix} -2 & -8 \\ -3 & -6 \end{bmatrix}$

At 16,0 Stable $A = \begin{bmatrix} -8 & -32 \\ 0 & -6 \end{bmatrix}$

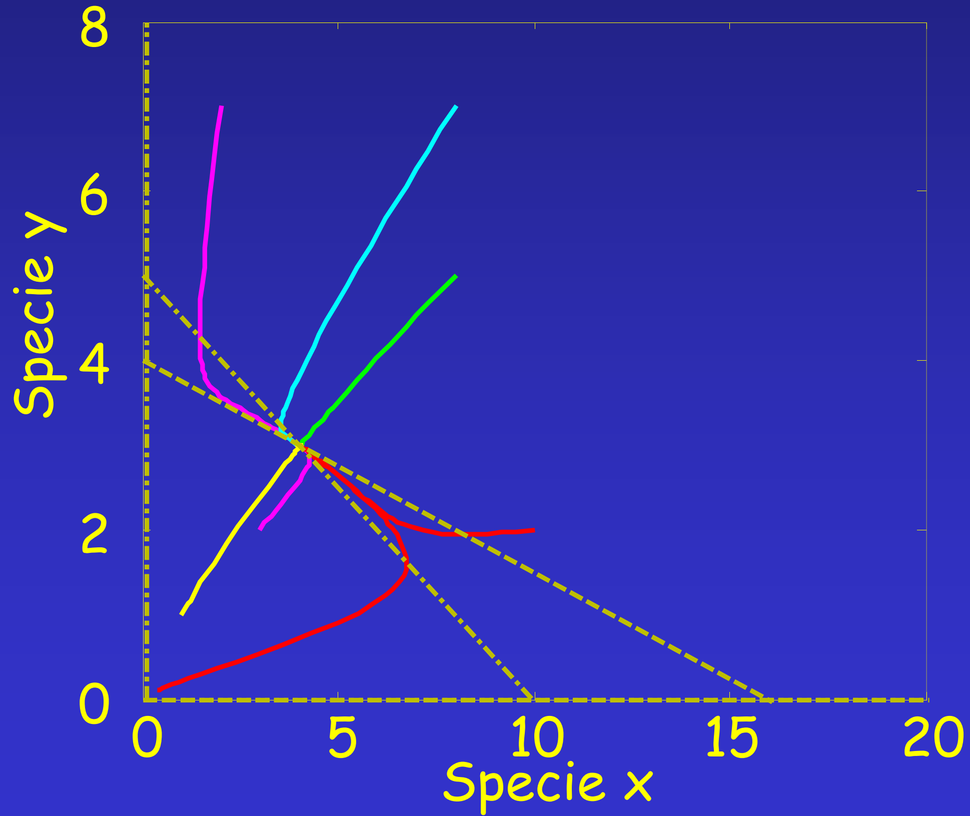
At 0,5 Stable $A = \begin{bmatrix} -2 & 0 \\ -5 & -10 \end{bmatrix}$

At 0,0 Unstable $A = \begin{bmatrix} 8 & 0 \\ 0 & 10 \end{bmatrix}$

Separatrix thru 4,3 – whether go to 16,0 or 0,5

# Competitive But Stable EQ – swap lines

$$\frac{dx}{dt} = x(10 - x - 2y);$$
$$\frac{dy}{dt} = y(8 - 0.5x - 2y)$$




At 4,3 $\quad \boldsymbol{A} = \begin{bmatrix} -4 & -8 \\ -1.5 & -6 \end{bmatrix}$

$\lambda = -1.394 \ \& \ -8.606$ stable

At 10,0 $\quad \boldsymbol{A} = \begin{bmatrix} -10 & -20 \\ 0 & 3 \end{bmatrix}$

$\lambda = -10$ and $3$ saddle

At 0,4 $\quad \boldsymbol{A} = \begin{bmatrix} 2 & -20 \\ -2 & -8 \end{bmatrix}$

$\lambda = -8$ and $2$ saddle

At 0,0 $\quad \boldsymbol{A} = \begin{bmatrix} 10 & 0 \\ 0 & 8 \end{bmatrix}$ u/s

No competition, x = 10, y = 4;
with, stable at x = 4, y = 3

comp bad!






Cybernetics

# Summary

We have seen simple populations of models.

- Single species,
- Interacting mutualists, predator-prey and competitors
  - Isoclines, where one pop stable, interact at equilibrium points.
  - These can be stable sink, unstable source, saddle, stable spiral, unstable spiral or centre
  - And that the state can be found from the eigenvalues of the Jacobean matrix for each point.
  - Mutualists – stable point larger pop than if no interaction
  - Competitors – if stable smaller pop than if no interaction.

This leads to considerations of attractors, which leads to fractals

# 3 : More Modelling

In this lecture we build on population modelling

Looking at attractors,

Discrete models – with recurrence

See some effects of self similarity

This then leads to fractals for artificial life

# *Emergence*

We wish for soft artificial life to emerge in a computer – so create a system that changes states over time:

A *phase space* (cf mathematics) is a space in which all possible states of a system are represented. Each possible state of the system corresponds to one unique point in the phase space

A *state space* representation (cf control engineering) is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations.

Can solve these to look at transient behaviour

But for A-Life more interested in steady state

This is Long-term behaviour : characterised by *Attractors.*

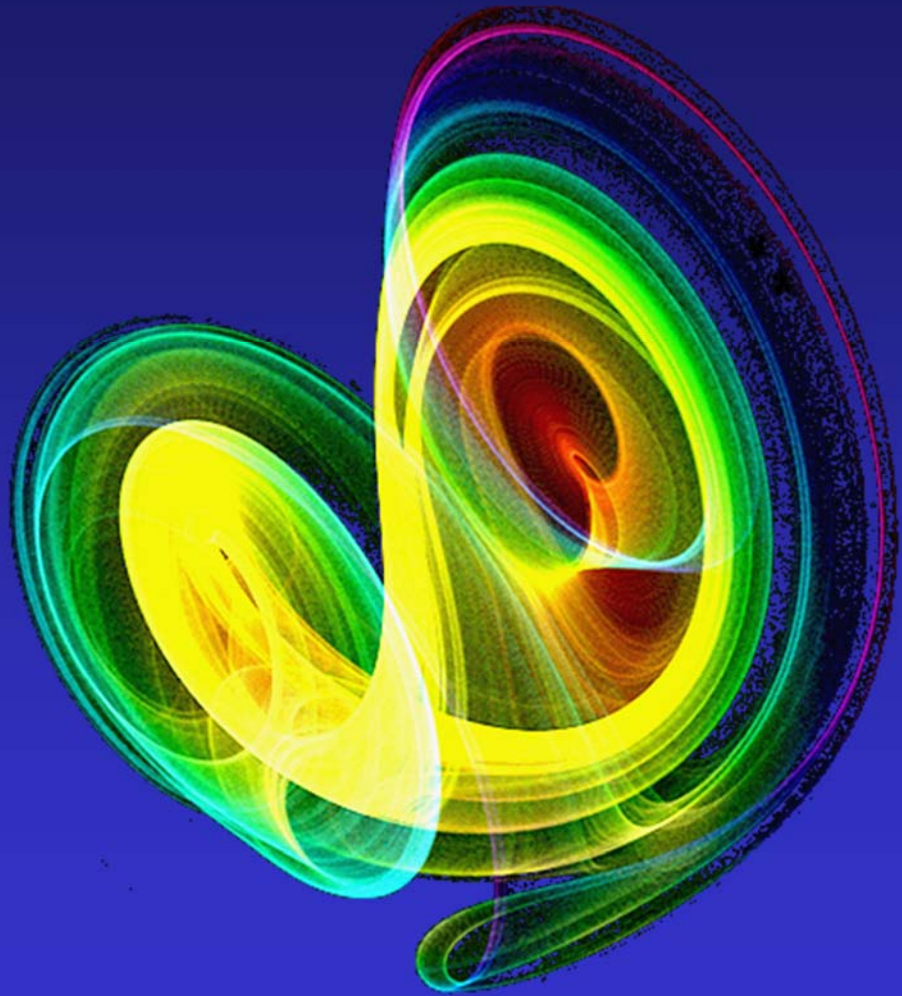
This relates to the equilibria we saw last time.

# Attractors

An attractor is a 'set', 'curve', or 'space' that a dynamical system irreversibly evolves to if left undisturbed .

May be known as a 'limit set'

Not necessarily trivial



# *Dynamics of Attractors*

Activation of each system unit is associated with direction in a multidimensional space (configuration space)

Every point in the space represents a possible state of the system (state vector).

Motion of this vector represents its evolution in time (described as a *dynamic system*)

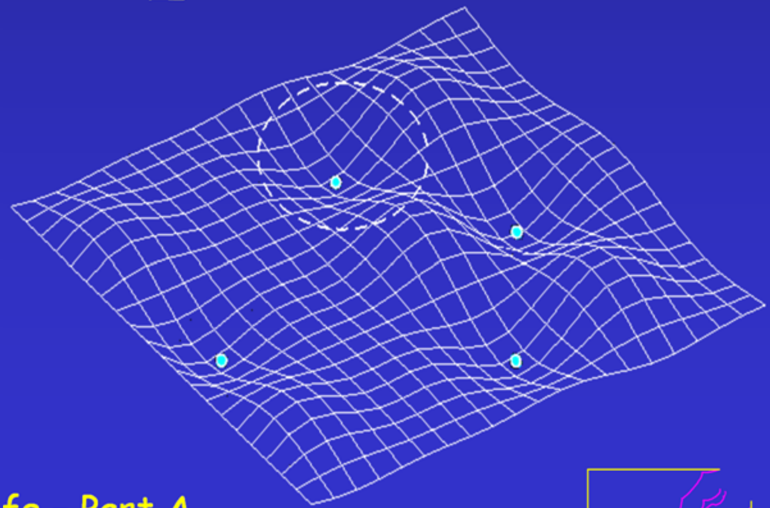
*http://www.scholarpedia.org/article/Attractor_network*

There are three types of attractors;

point attractors,

periodic attractors

strange attractors






Cybernetics

# *Why use Attractors?*

No need to know the start time or duration or to observe entire evolution of system to get result

Only need to know the attractor, a given initial condition involves a path towards an attractor and an indication that the system has reached the attractor

They allow control of timing of system's response

Robust to small changes in system parameters

- dilution (removal of system connections)
- asymmetry
- clipping/quantization of parameter values

NB system could be 'computer' based (e.g. networks), biological (e.g. heart) or mechanical (e.g. pendulum).

# Using Attractors

These systems are typically defined as series of ODEs

$$\frac{dx}{dt} = 10(y-x)$$

$$\frac{dy}{dt} = -xz + 28x - y$$

$$\frac{dz}{dt} = xy - \frac{8}{3}y$$

*http://www.edc.ncl.ac.uk/highlight/rhnovember2006g02.php/*

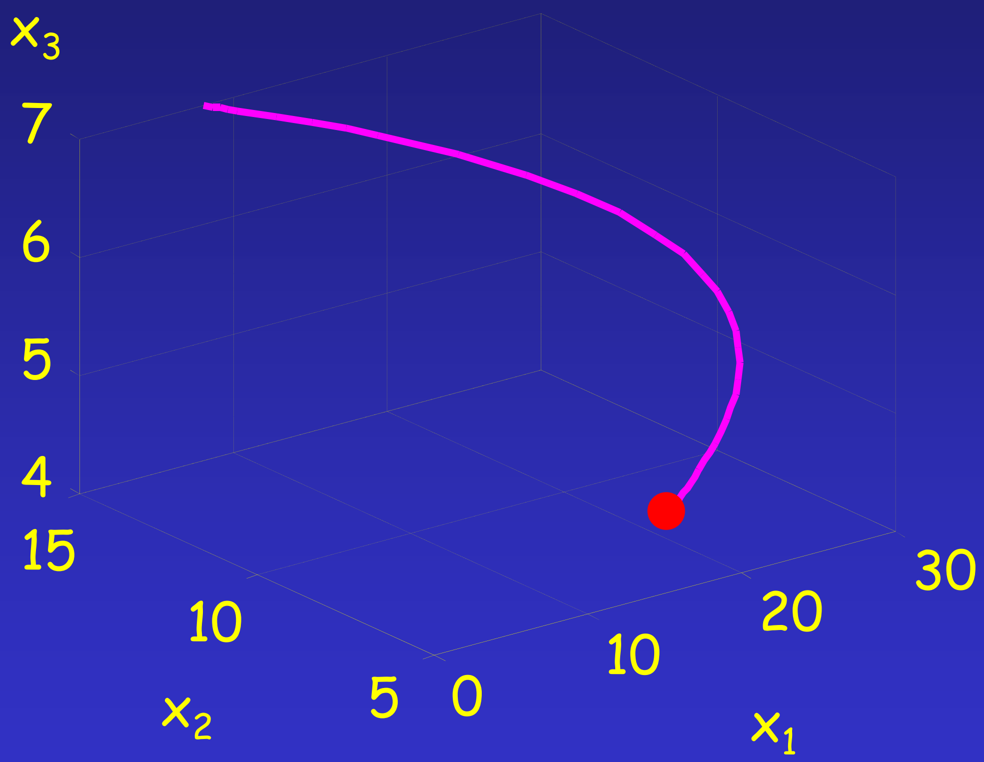
Computation performed by mapping from an initial condition to a particular attractor

Dynamics partition the configuration space into basins of attraction around the attractors. Let's see some.

# Fixed Point Attractor

The state vector comes to rest

- Results are computed as different input data settle into different fixed points
- The region of initial states that settle into a single fixed point is called its basin of attraction
- Most networks are fixed point networks
- C.f. Stable equilibrium point in population examples




Example with three variables

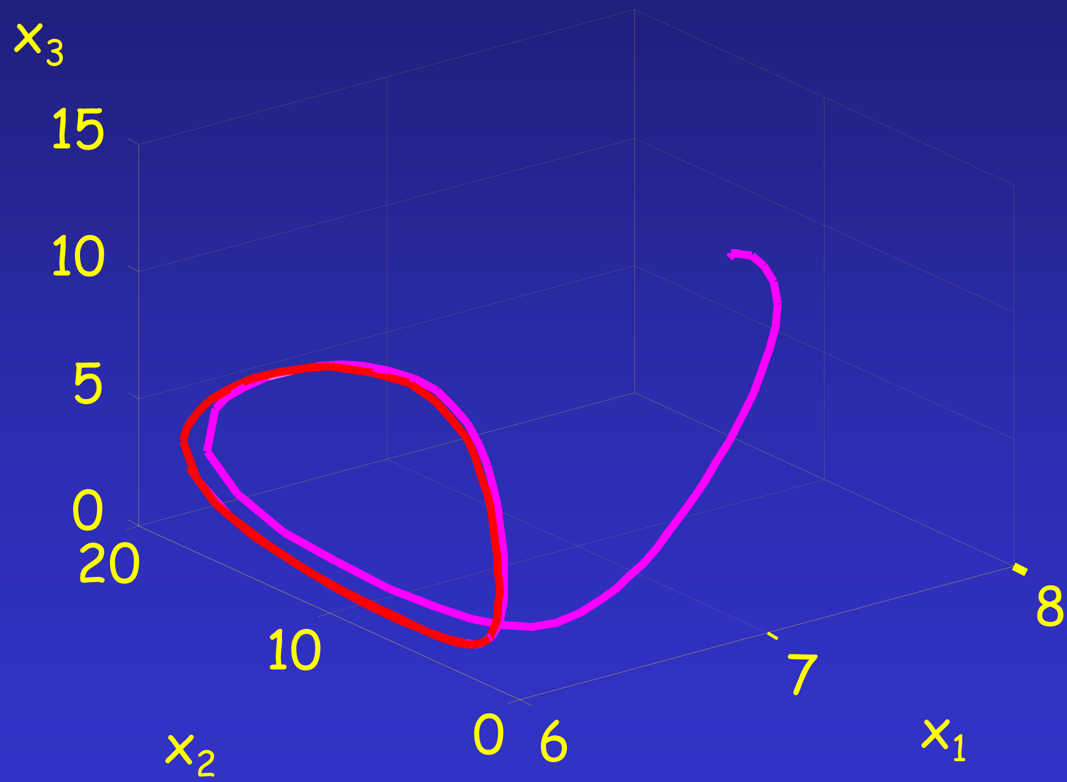
# *Limit Cycle Attractor*

The state vector settles into a periodic cycle

The attractor is a limit cycle

-Shown in red

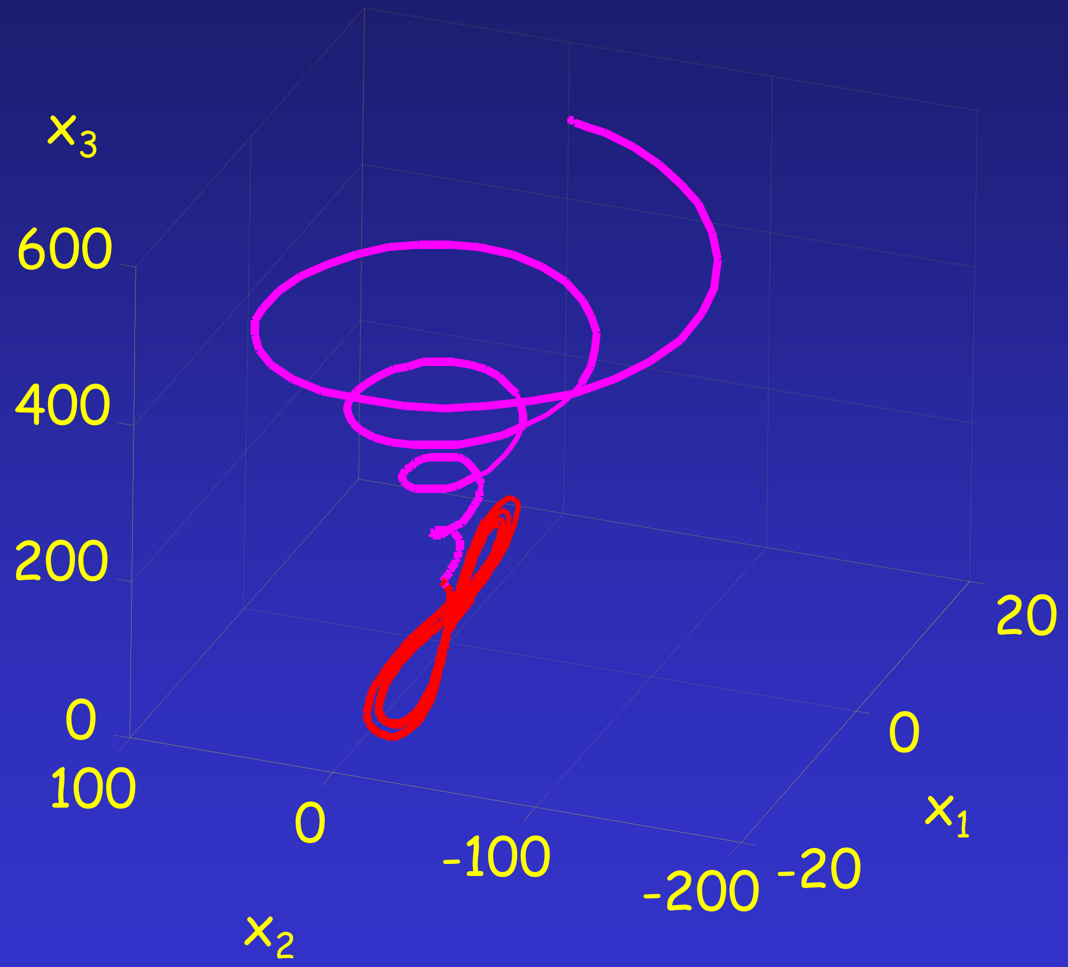
Transient in purple



# Strange Attractor - chaotic

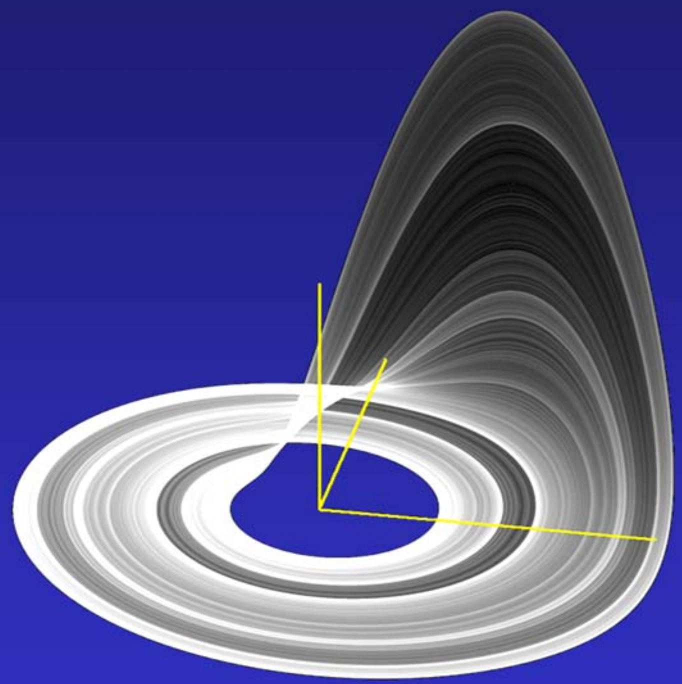
Two copies of the system that initially have nearly identical states will grow dissimilar as they evolve.

Divergence is restricted so that in many directions the state vectors are growing closer



# Higher Dimensions

chaotic attractors (also encounter repellors)



$$\frac{dy}{dt} = x + a\,y$$

$$\frac{dz}{dt} = b + z\,(x - c)$$

$$\frac{dx}{dt} = -y - z$$

Rössler studied chaotic attractor a = 0.2, b = 0.2, c = 5.7

NB a = 0.1, b = 0.1, and c = 14 more commonly used since

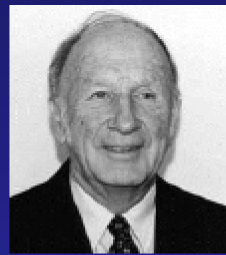
# *Lorenz attractor*

$$\frac{dx}{dt} = \sigma(y - x)$$

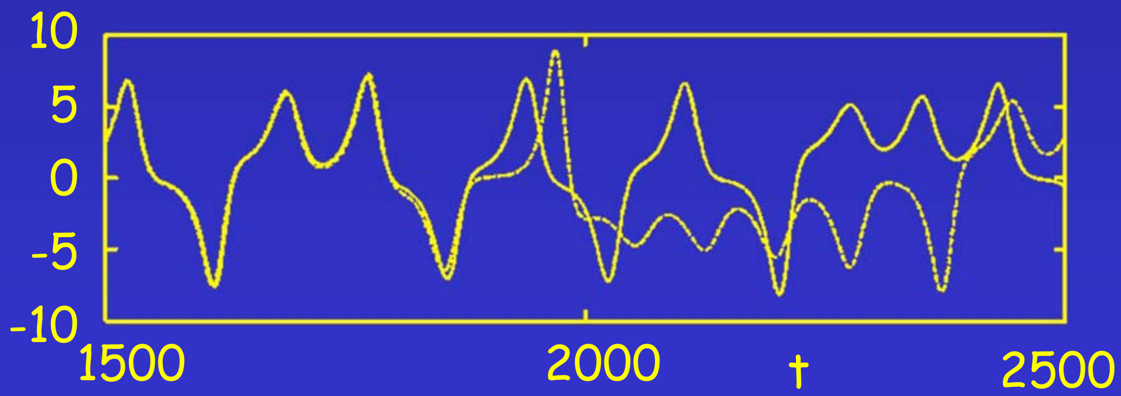
$$\frac{dy}{dt} = x(\rho - z) - y$$

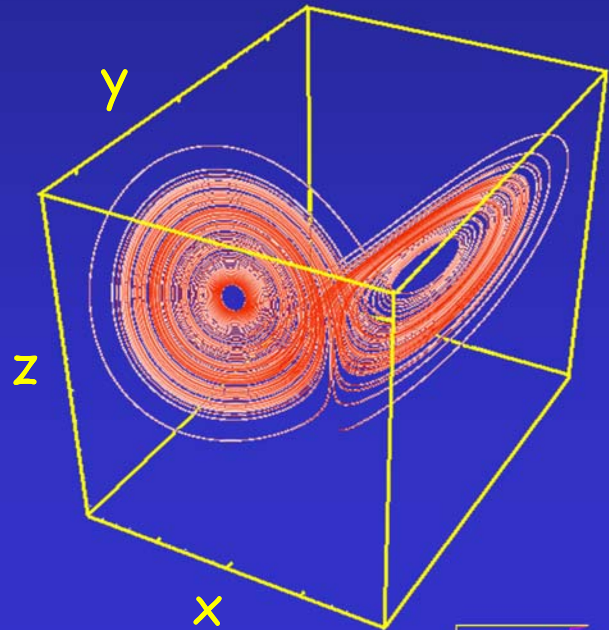
$$\frac{dz}{dt} = xy - \beta z$$

$\sigma$ is called the Prandtl number,
$\rho$ is called the Rayleigh number.
All $\sigma, \rho, \beta > 0$, but usually
$\sigma = 10$, $\beta = 8/3$ and $\rho$ is varied



Simple model of convection in atmosphere.
Sensitive to initial conditions



# Some Discrete Models

In the above, the models were continuous

They can be of single species, or multiple.

We now move to considering discrete models

Here $x_n$ is the 'population' at 'time' n

The change in $x_n$ is set by a recurrence relation of the form

$$x_{n+1} = r\, x_n\, (1 - x_n)$$

These are of interest as the value of r affects what happens

Different steady states are found

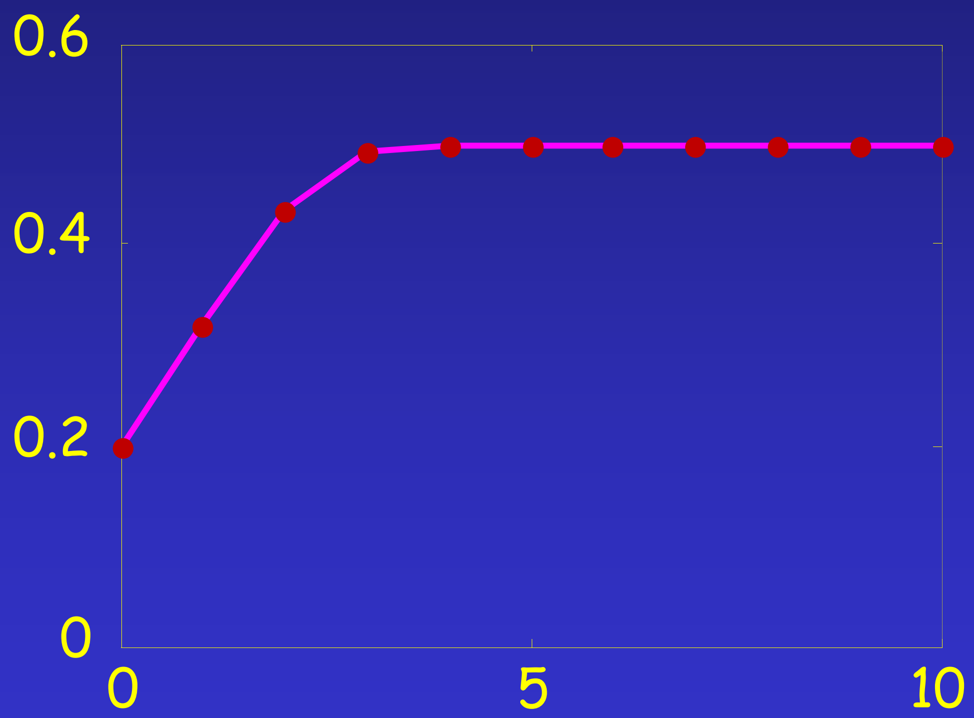
These show self similarity, which leads nicely to fractals.

# Logistic Map – discrete model

$x_{n+1} = r\, x_n\, (1 - x_n)$

| n | x |
|---|---|
| 0 | 0.2000 |
| 1 | 0.3200 |
| 2 | 0.4352 |
| 3 | 0.4916 |
| 4 | 0.4999 |
| 5 | 0.5000 |
| 6 | 0.5000 |
| 7 | 0.5000 |
| 8 | 0.5000 |
| 9 | 0.5000 |
| 10 | 0.5000 |

$x_0 = 0.2;\ r = 2$

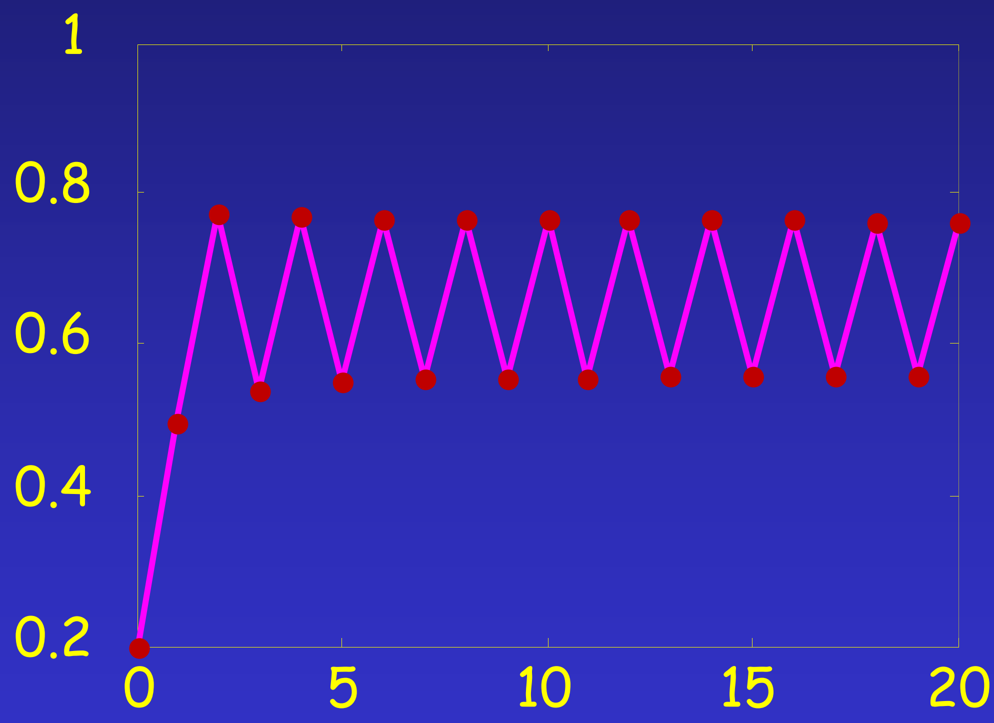


# *But if r changed to 3.1*

$x_{n+1} = r\, x_n\, (1 - x_n)$

| n | x |
|---|---|
| 0 | 0.2000 |
| 1 | 0.4960 |
| 2 | 0.7750 |
| 3 | 0.5406 |
| 4 | 0.7699 |
| … | |
| 21 | 0. 7646 |
| 22 | 0.5579 |
| 23 | 0.7646 |
| 24 | 0.5579 |

$x_0 = 0.2; r = 3.1$



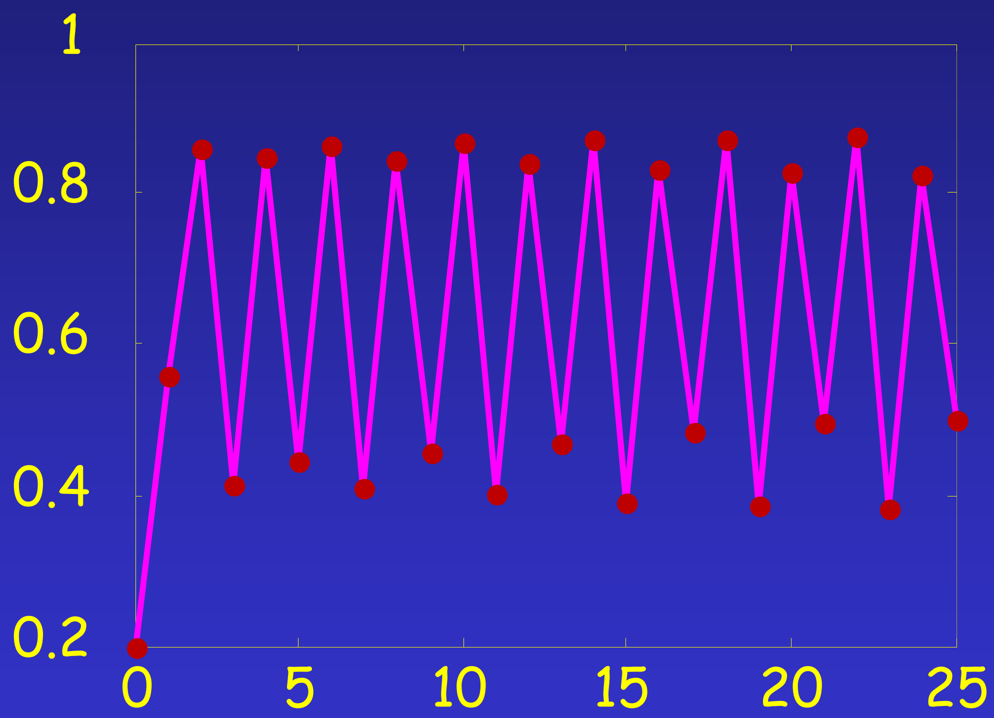

Finally – oscillates between 2 values

# *But if r is 3.5 …*

$x_{n+1} = r\, x_n\, (1 - x_n)$

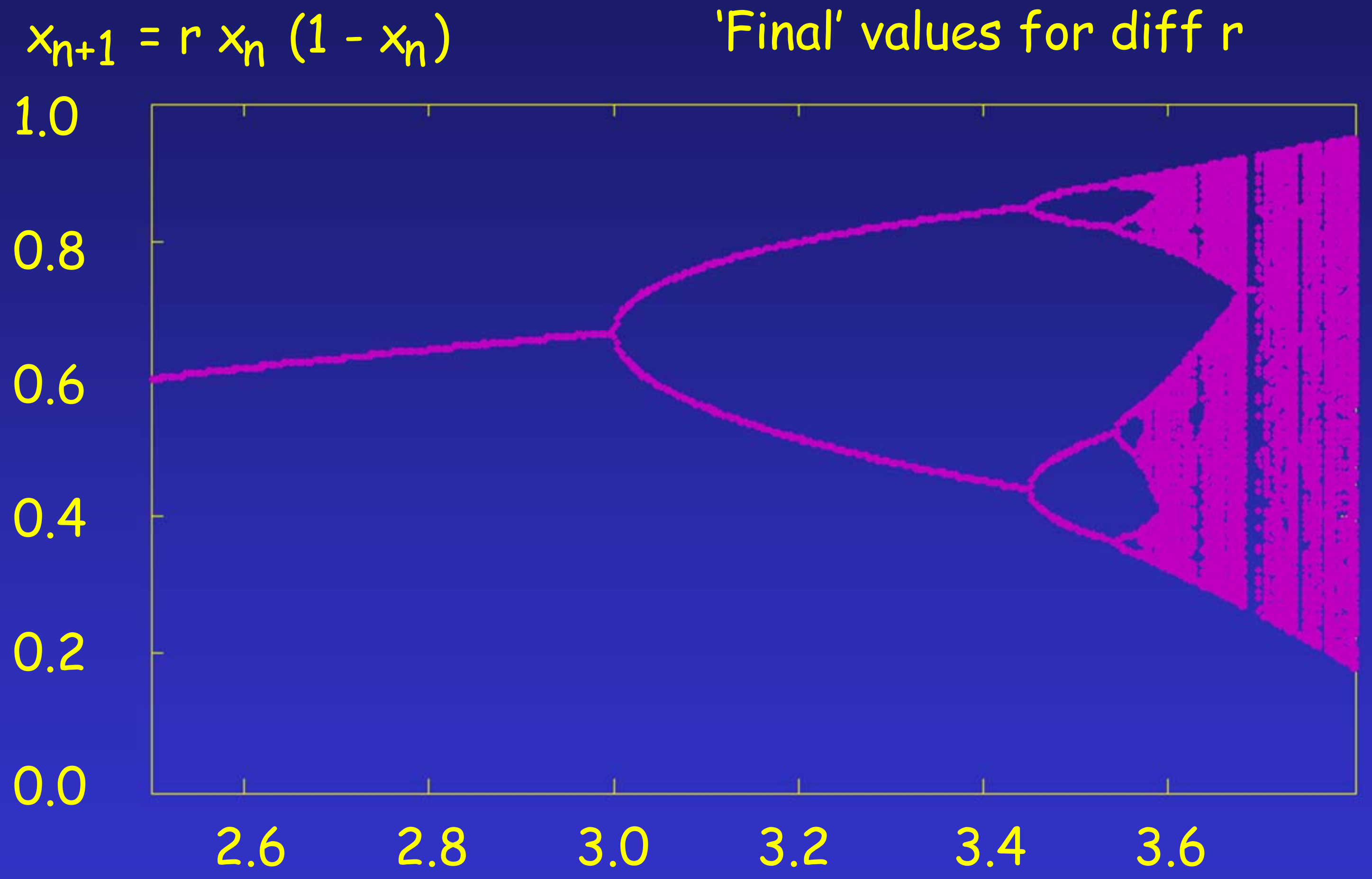
$x_0 = 0.2;\ r = 3.5$

| n | x |
|---|---|
| 0 | 0.2000 |
| 1 | 0.5600 |
| 2 | 0.8624 |
| 3 | 0.4153 |
| 4 | 0.8499 |
| … | |
| 27 | 0.8750 |
| 28 | 0.3828 |
| 29 | 0.8269 |
| 30 | 0.5009 |
| 31 | 0.8750 |




Oscillates between 4 values

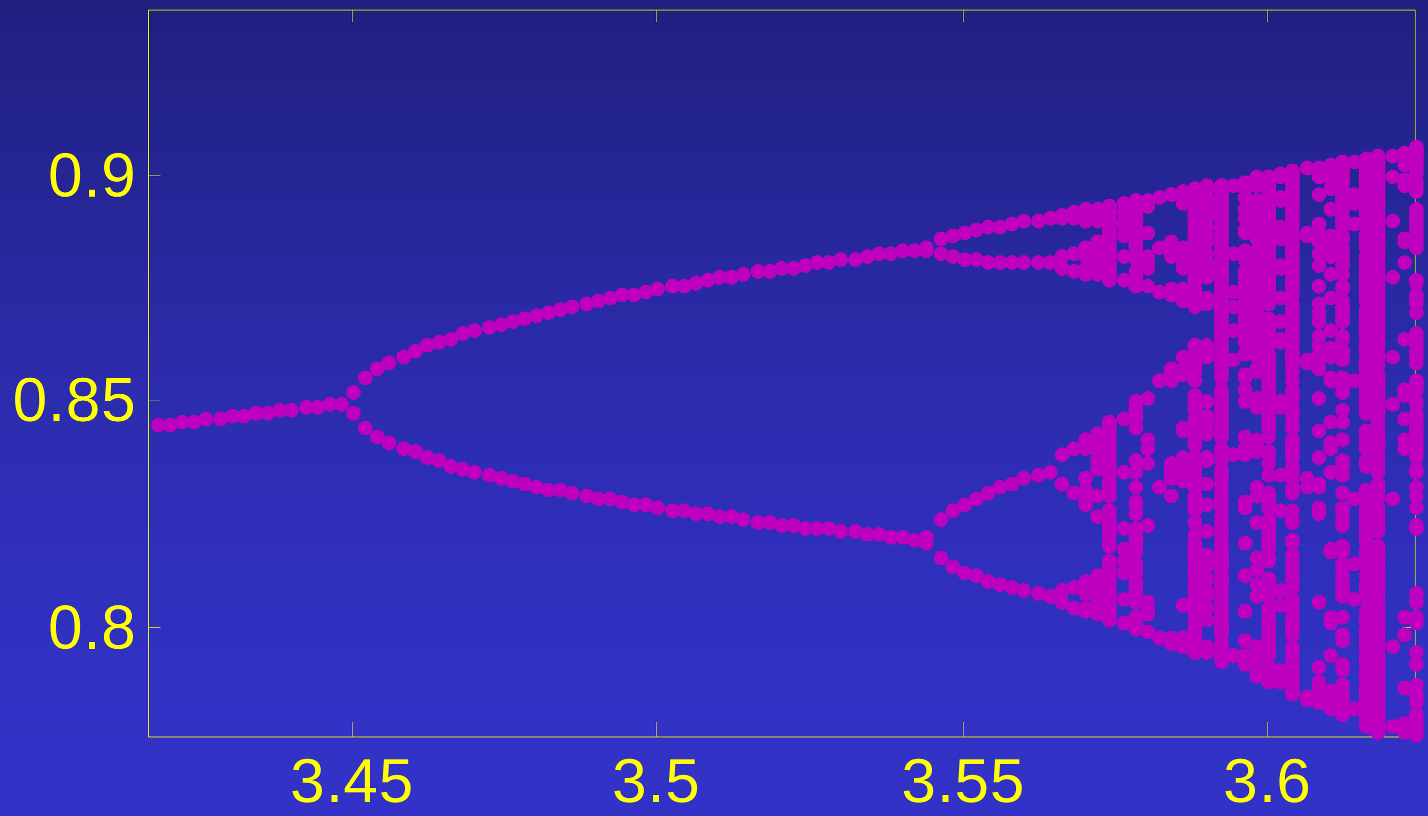
# 'Final' values for diff r




*See also http://en.wikipedia.org/wiki/File:LogisticCobwebChaos.gif*

# Self-similarity

Compare prev with when zoom in ..



# *Orbits*

The Hénon map is a discrete-time dynamical system - exhibiting chaotic behavior. The Hénon map takes a point (x, y) in the plane and maps it to a new point

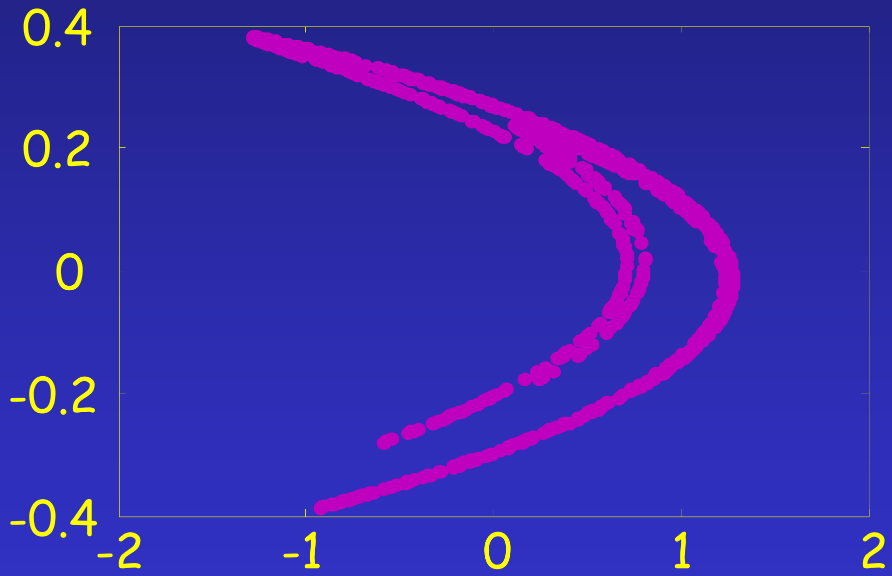
$$x_{n+1} = y_n + 1 - a x_n^2$$

$$y_{n+1} = b x_n$$

Here a = 1.3 b = 0.4

Known to be chaotic

Orbit – plots of x,y




For other values of a and b the map may be chaotic, intermittent, or converge to a periodic orbit

# *Self Similarity - Fractals*

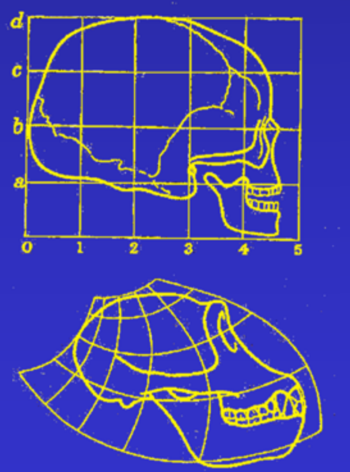
The self similarity observed earlier, leads to Fractals

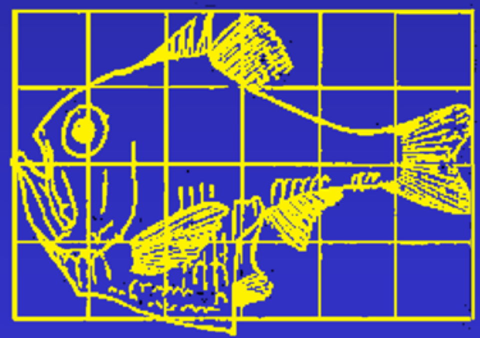
These have been used in various ways re modelling life

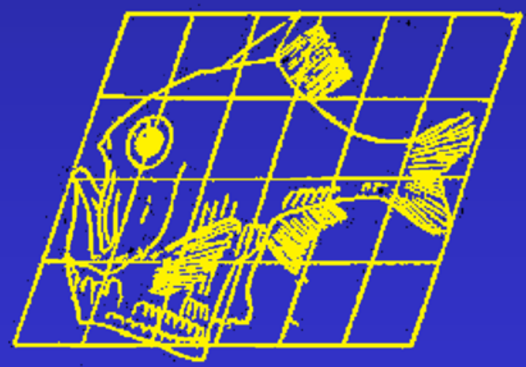
Examples include trees, Plants, Clouds, Mountains

D'arcy Wentworth Thompson, On Growth and Form, 1917

Laid foundations for biomathematics; found equations to describe static forms of organisms; saw transformations by changing paras





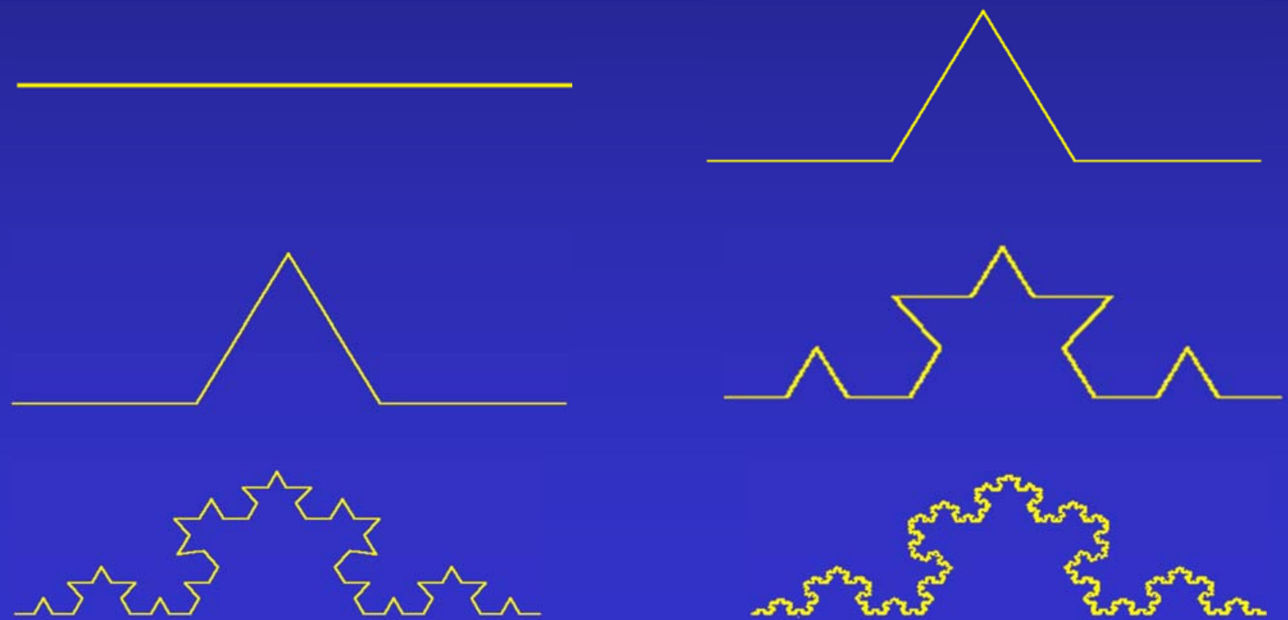
# Fractals

Complex objects defined by systematically and recursively replacing parts of a simple start object with another, using a simple rule

Simplest : Have initiator and generator, both many lines.

Replace each line in the initiator with the generator shape.

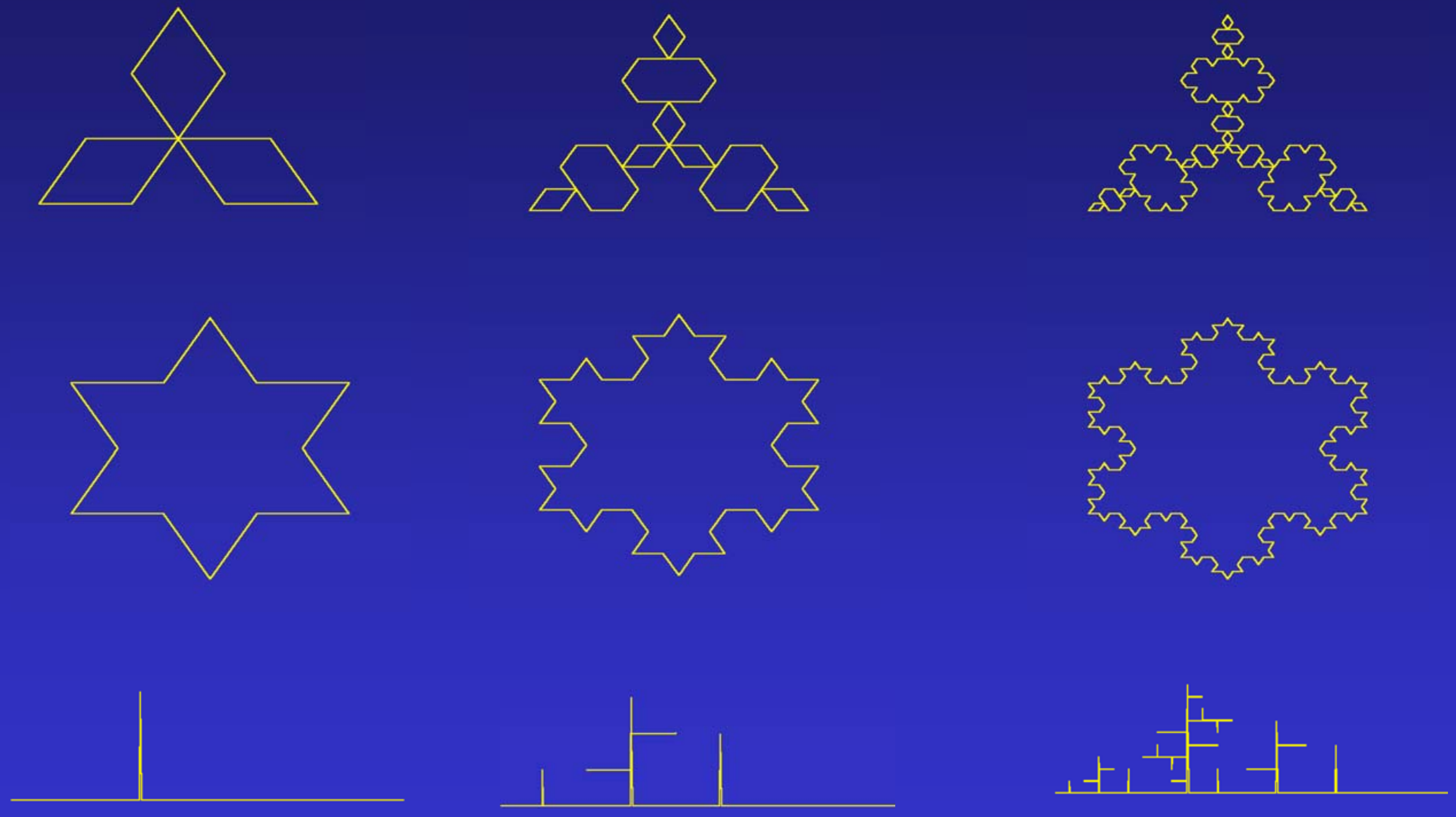
Makes more lines, so replace all these lines with generator



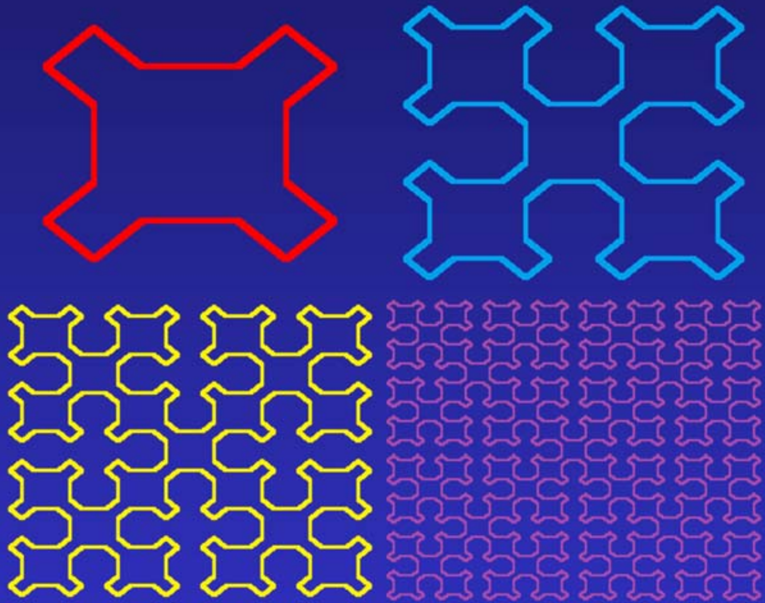


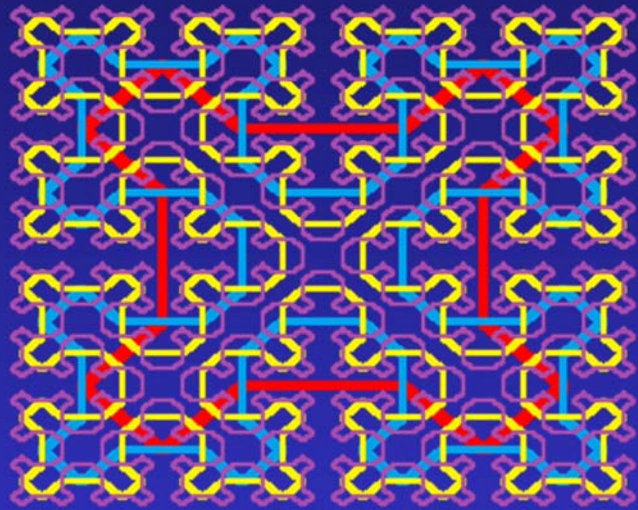

Cybernetics

# Koch, SnowFlake, Forest Examples



# Sierpinski : Space Filling Curve



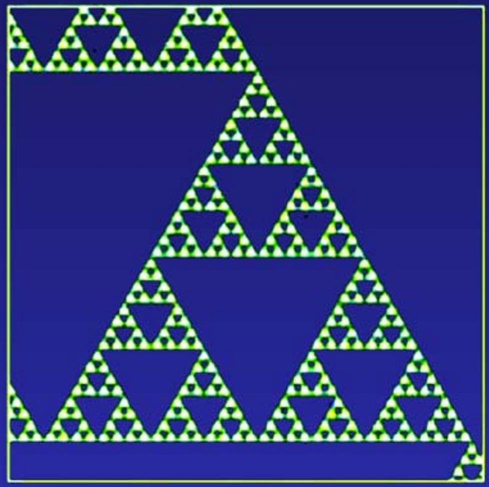


four shapes _/ (+ rotations) A B C D, joined by four corners.

A(n) is A(n-1) \ B(n-1) _ D(n-1) / A(n-1) {n > 0}

A(0) is nowt. Similar for B(n), C(n) and D(n)

# Also



Sierpinski Gasket :
triangle from which
smaller triangles are cut

Can also get
'natural' fractals ..

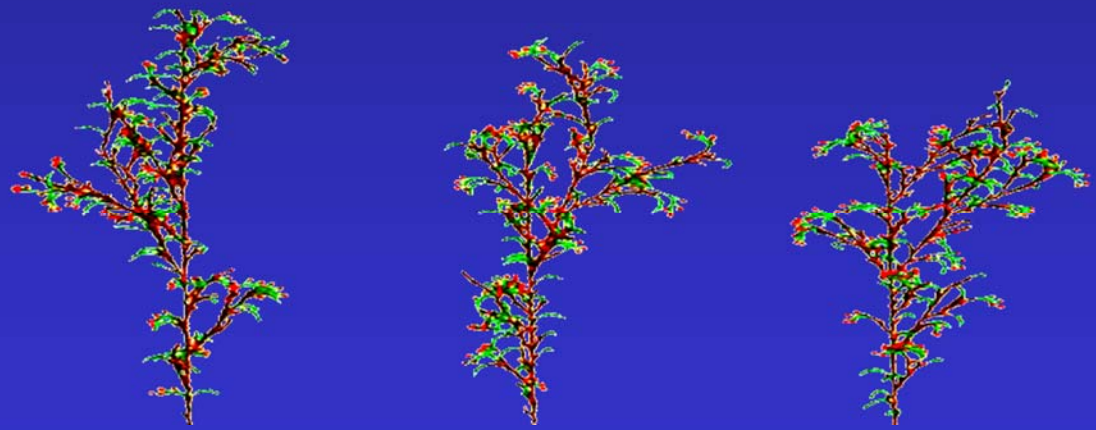
More sophisticated replication methods …

# *Lindenmayer System (L-Systems)*

Mathematical formalism proposed by biologist Aristid Lindenmayer in 1968 : foundation for axiomatic theory of biological development

A Lindenmayer system is a variant of a formal grammar (a set of rules and symbols), acting as a parallel rewriting system

It models the growth processes of plants, organisms and self-similar fractals – due to the recursive nature of the rules.



Useful: http://algorithmicbotany.org/papers/#abop

# Details

L-systems are defined as a tuple

$$G = \{V, S, \omega, P\}$$

where

$V$ (the alphabet) is a set of symbols containing elements that can be replaced (variables)

$S$ is a set of symbols containing elements that remain fixed (constants)

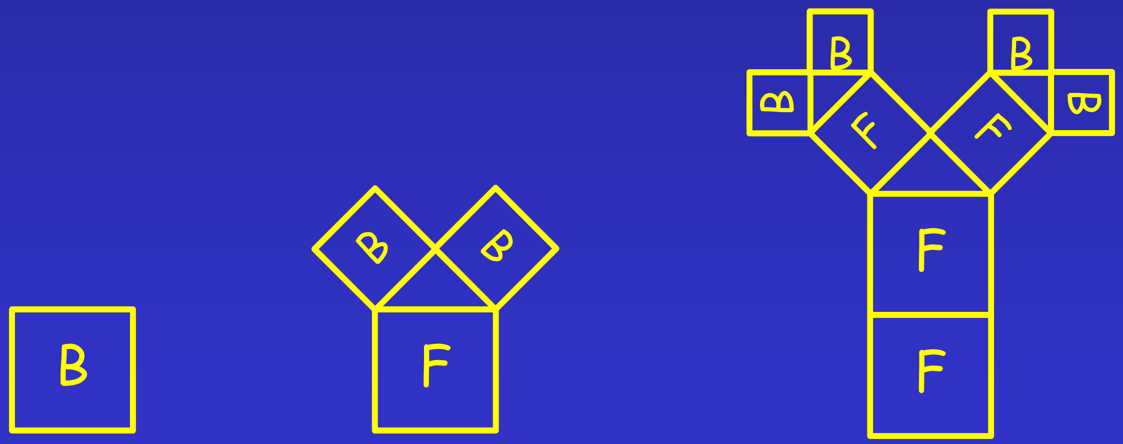
$\omega$ (start, axiom or initiator) is a string of symbols from V defining the initial state of the system

$P$ is a set of production rules defining the way variables can be replaced with combinations of constants and other variables.

# *Example*

An L-system is an ordered triplet

- *G = <V, w, P>*
- *V = alphabet of the symbols in the system;* V = {F, B}
- *w = nonempty word* , the axiom: B
- *P = finite set of production rules (productions)*
- B := F[-B][+B]
- F := FF



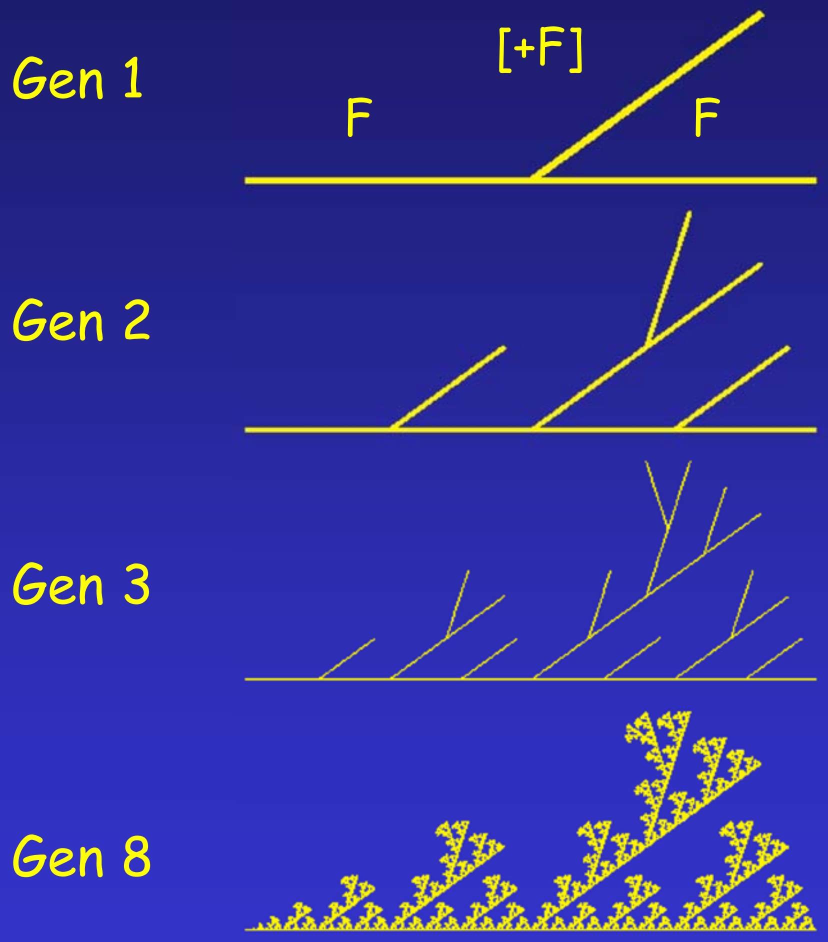
# Production Rules for Artificial Plants

Add branching symbols [ ]

simple example

Main trunk shoots off one side branch

- Angle 45
- Axiom: F
- Seed Cell
- Rule: F=F[+F]F
- Angle



# *Some Examples*

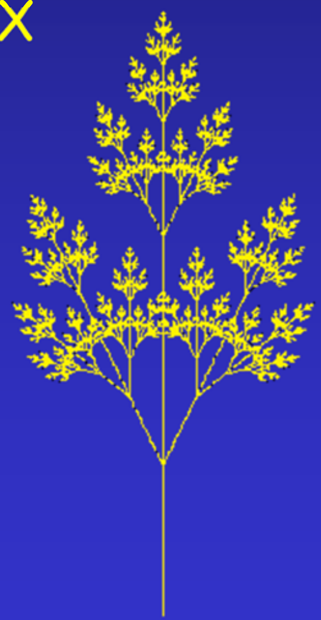
V = {F, X}        the alphabet

the axiom: X

*P = finite set of production rules*

X := F[+X][-X]FX

F := FF

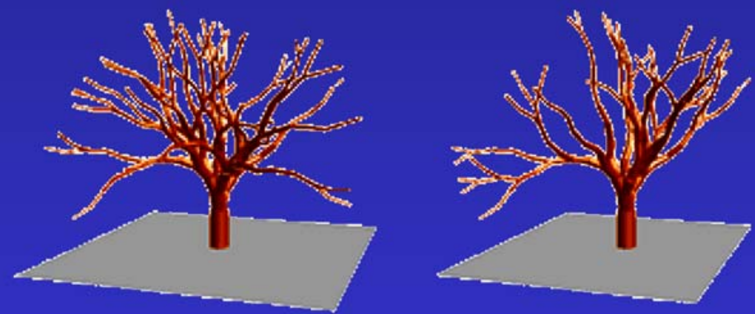


Probabilistic production rules

*A* := B *C* ( P = 0.3 )

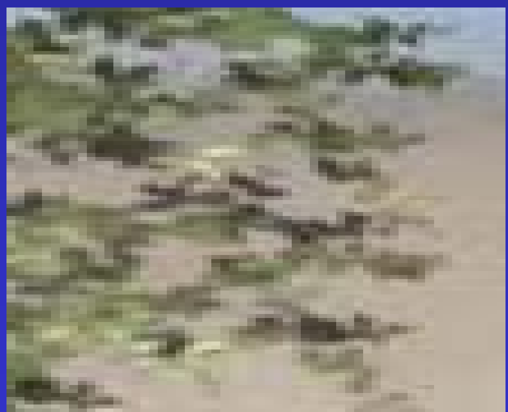
*A* := F *A* ( P = 0.5 )

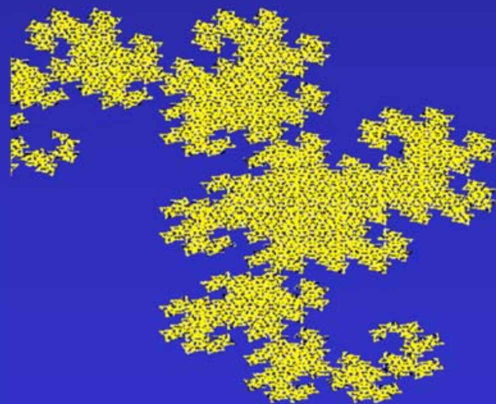
*A* := *A* B ( P = 0.2 )



http://coco.ccu.uniovi.es/malva/sketchbook/

# More Example L - Systems





Colin McRae Dirt : pre-generated and preloaded!

# Making Life Realistic

Fractals etc can help make realistic looking images

But 'life' tends to move and interact

So want artificial life to also behave realistically

Need to define appropriate behaviour, dependent on surroundings

Of interest is having situations with multiple entities

Applications

- Film and TV
- Games
- Simulations for engineering, architecture and transport

Premier system is MASSIVE …

# MASSIVE

Software package from Stephen Regelous for visual effects

Key feature : can create 1000s …1000000s of agents

Fuzzy logic used so each agent react individually to surroundings

Used to control prerecorded animation clips

(say from motion capture or hand animation)

Creates characters that move, act and react realistically

Developed initially for Lord of the Rings …

Used in Avatar, King Kong, Narnia, I Robot, Doctor Who, WallE, …

http://www.massivesoftware.com/

# Some Images



# Summary

We have looked at more modelling of systems

Some differential equations, and the associated attractors which define their steady state

We have considered discrete models – recurrence relations, and seen the different states, and the associated self similarity

This lead to fractal systems, including Lindenmayer systems, which have been used to produce computer generated images

More sophisticated examples also exist, of 'agents' interacting with others, determining their actions/ movements.

Next time, we move to hardware systems and how they learn.