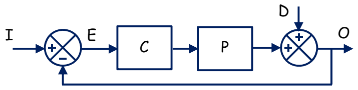
6 : Feedback Not Just for Control

We have modeled a feedback control system by the diagram:



We have used 'forward over one minus loop' rule to show that

If D = 0, $\frac{O}{I} = \frac{C * P}{1 + C*P}$ or $O = \frac{C * P}{1 + C*P} * I$

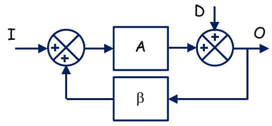
If I = 0, $\frac{O}{D} = \frac{1}{1 + C*P}$ or $O = \frac{1}{1 + C*P} * D$

If C*P is large, O ~ I + 0 * D …

What if want O bigger than I?

Control Engineers want O = I, Audio Engineers O = I * G

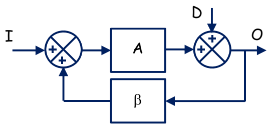
More General Feedback System has this form:

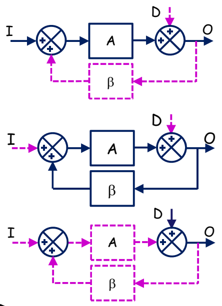


We can again analyse by forward over one minus loop

NB this is Control system if $A = C * P$, $\beta = -1$!

Analysing this Aβ System





If D = 0, $\frac{O}{I} = \frac{\text{Forward}}{1 - \text{Loop}} = \frac{A}{1 - A\beta}$

If I = 0, $\frac{O}{D} = \frac{\text{Forward}}{1 - \text{Loop}} = \frac{1}{1 - A\beta}$

By Superposition:

$$O = \frac{A}{1 - A\beta} I + \frac{1}{1 - A\beta} D$$

With Some Numbers

Suppose Aβ >> 1 {much greater than 1} 'negligibly large', 1 – Aβ ~ Aβ

$\frac{O}{I} = \frac{A}{1 - A\beta} \approx \frac{A}{-A\beta} = -\frac{1}{\beta}$ (O is independent of A)

e.g. if A = -5000, β = -0.2, Aβ = 1000, 1-Aβ = -999 ~ -Aβ

So $O \approx -\frac{1}{\beta} * I = 5 * I$ Actually $O \approx \frac{-5000}{-999} * I = 5.005 * I$

Also $O = \frac{1}{-999} * D = -0.001*D \approx 0*D$

If $A \to 5050$, $O \approx \frac{-5050}{-1009} * I = 5.005 * I$

Also works ..

Also works if Aβ << -1, (large and negative)

e.g. if A = -5000, β = 0.2, Aβ = -1000, 1 - Aβ = 1001, ~ - Aβ

O = -4.995 * I ~ -1/β * I, independent of A and O ~ 0*D

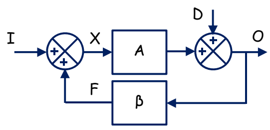
Feedback **good** if modulus of Loop Gain, |Aβ |, large

{ modulus means size irrespective of sign: | 5 | = 5 |-5| = 5 }

Then O = I times -1/feedback value, is independent of A (and hence of changes in A) and unaffected by D.

If loop gain smaller, result not as good …

Example – A = 100, β = 0.21



1 - Aβ = 1-21 = -20

$\frac{O}{I} = \frac{A}{1 - A\beta} = \frac{100}{-20} = -5$

$\frac{O}{D} = \frac{1}{1 - A\beta} = \frac{1}{-20} = -0.05$

If I = 2, and D = 0, O = -5*2 = -10

Check: F = -10*0.21 = -2.1, so X = 2+-2.1 = -0.1; O = 100*-0.1+0 = -10

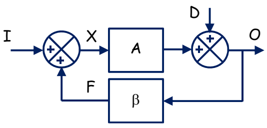
If I = 0 and D = 1, O = -0.05 * 1 = -0.05.

Check: F = -0.0105 = X, so O = -0.0105*100+1 = -0.05

If I = 2 and D = 3; O = -5*2 + 3*-0.05 = -10.15

Check: F = -2.1315; X = -0.1315; O = -13.15+3 = -10.15

Lecture 6 - In Class Exercise



Suppose A = 200, β = -0.2

a) Find O/I if D = 0

b) Find O/D if I = 0

c) Find %change in O/I if A changes by 10% to 220

Positive/Negative Feedback

We will correct erroneous definitions / claims often made.

Wrong to say negative feedback because - sign in 'summer'

(Changing sign of β has same effect as changing + to -)

The important point is to have both

a) claims for what negative feedback does

b) a consistent definition for negative feedback

To that end the correct view is that Negative Feedback

a) reduces effects on output of disturbances

reduces effects on output of parameter changes

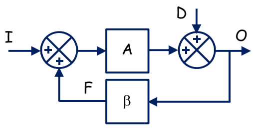
b) occurs if |closed loop gain| < |open loop gain|

NB | x | or modulus of x, means size : ignore sign

Negative Feedback (Harold Black 1930s)

Forward (Open Loop) Gain = A

Closed Loop Gain = $\frac{A}{1 - A\beta}$



Negative Feedback

| Closed Loop Gain | < | Open Loop Gain |

$\left|\frac{A}{1 - A\beta}\right| = \frac{|A|}{|1 - A\beta|} < |A|$ i.e. $\frac{1}{|1 - A\beta|} < 1$ or $1 < |1 - A\beta|$

Negative Feedback, when | 1 – Aβ | > 1,

Reduces effect of Disturbances D on output O

Reduces effect of changes in A on output O

Negative Feedback & Disturbances

Open Loop: $\frac{O}{D} = 1$; no reduction in effect of D

Closed Loop : $\frac{O}{D} = \frac{1}{1 - A\beta}$; reduction if $|1 - A\beta| > 1$

A = 5 and β = -4: $1 - A\beta = 1 + 20 = 21$. Negative Feedback.

$\frac{O}{D} = \frac{1}{21} < 1$ D must change by 21 if change O by 1

A = 5 and β = 0.04: $1 - A\beta = 0.8 < 1$. Positive Feedback

$\frac{O}{D} = \frac{1}{0.8} = 1.25$ D in effect amplified

… and Changes in Parameters

Let A change by a <u>small</u> proportion: call it δ: i.e. A := A(1 + δ)

Relative change in open loop = $\frac{A(1+\delta)-A}{A} = \delta$

Relative change in closed loop (see next slide) = $\frac{\delta}{1-A\beta}$

Feedback reduces the effect of change in A if |1 - Aβ | > 1.

Let A = 5 and β = -4 (-ve fb) and A change by 10% to 5.5 ie δ = 0.1

Rel Change: open loop = 0.1; closed loop = 0.1/21 = 0.005 (smaller)

If instead β = 0.04 (+ve fb)

Rel Change: open loop = 0.1; closed loop = 0.1/0.8 = 0.125 (bigger)

Interest Only: A → A(1 + δ)

Closed Loop Gain = $\frac{A(1+\delta)}{1-A(1+\delta)\beta}$ Relative change = $\frac{\text{new - old}}{\text{old}}$

$$\frac{\frac{A(1+\delta)}{1-A(1+\delta)\beta} - \frac{A}{1-A\beta}}{\frac{A}{1-A\beta}} = \frac{(1+\delta)(1-A\beta)}{1-A(1+\delta)\beta} - 1$$

$$= \frac{(1+\delta)(1-A\beta) - (1-A(1+\delta)\beta)}{1-A(1+\delta)\beta}$$

$$= \frac{1+\delta \quad - A\beta - \delta A\beta - 1 \; + \; A\beta \; + \; \delta A\beta}{1-(A+A\delta)\beta} = \frac{\delta}{1-A(1+\delta)\beta}$$

As $\delta << 1$, this approximates to $\frac{\delta}{1 - A\beta}$, as stated earlier

Can also do by differentiating closed loop TF w.r.t A

Effect of Changing A : 50 to 55

β = a) -1/50 b) -1/10 c) 1/100 d) 3/100 e) 1/10

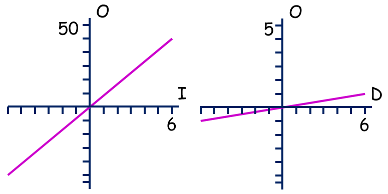
	β	1-Aβ	$\frac{O}{D} = \frac{1}{1-A\beta}$	$\frac{O}{I} = \frac{A}{1-A\beta}$	$\frac{O}{I}$ (A = 55)	% diff
a	-1/50	1+1=2	1/2=0.5	50/2=25	55/2.1= 26	+4%
b	-1/10	1+5=6	1/6=0.17	50/6=8.3	55/6.5= 8.5	+1.5%
c	1/100	1-0.5=0.5	1/0.5=2	50/0.5=100	55/0.45= 122	+22%
d	3/100	1-1.5=-0.5	1/-0.5=-2	50/-0.5=-100	55/-0.65= -84.6	-15%
e	1/10	1-5=-4	1/-4=-0.25	5 -/-4=-12.5	55/-4.5= -12.2	-2%

b) highest loop gain: best at rejecting D & changes in A.
No system has high loop gain.
c) and d) have +ve feedback: O/D > 1 and % diff > 10%

Graphs for b) : A = 50, β = -0.1

It can be useful to plot graphs of O vs I and O vs D

Here $\frac{O}{I} = \frac{50}{6}$ and $\frac{O}{D} = \frac{1}{6}$, so



Both straight lines thru 0,0
O/I thru
O = 50, I = 6
O/D thru
O = 1, I = 6

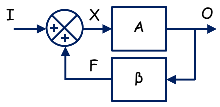
As straight lines … associated system is said to be ***linear***
If gradient of O/D < 1 : system has negative feedback

Summary

We have analysed simple feedback systems :
used forward over one minus loop for closed loop TF
We have seen benefit of high loop gain
We have considered positive and negative feedback.
Which applies to CP systems … test if |1 + CP | > 0

So far we assume that as I increases so will O
In practice this is not true, so next week
We consider what happens when systems are limited

Lecture 6 After Class Exercise

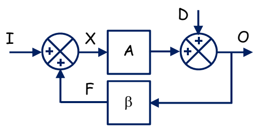


Here, I = 2, A = 10.
Find O and state whether positive feedback if:
a) β = -1
b) β = +1
c) β = -0.02
d) β = +0.02

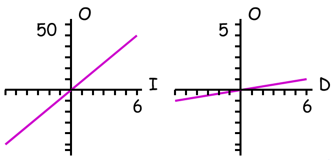
7 : Feedback Systems and Limits

Signals cannot be infinite … they are limited … we determine effect.

Recall last week's system with
A = 50, β = -0.1
1 - Aβ = 1+5 = 6



$\frac{O}{I} = \frac{50}{6}$ and $\frac{O}{D} = \frac{1}{6}$

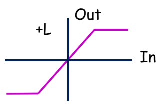


Straight line graphs – system is linear
|1 - Aβ| > 1 : -ve fb

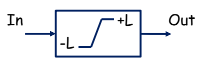
Limits

O/I graph implies that as I increases, so O increases
Not true in practical systems
e.g. output of a component can't exceed its power supply, etc
The output has limits; and we can incorporate them.

Below are shown limits graphically and as a block diagram

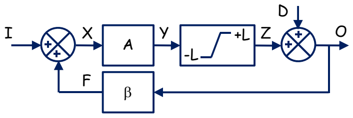


Limit Block



If -L ≤ In ≤ L, Out = In;
if In < -L, Out = -L; if In > L, Out = L

Limits in Feedback Systems



Let A = 50; β = -0.1; +L = +25 and -L = -25.

Consider O vs I assuming D = 0.

When no limiting, limit box transfer function = 1.

Thus $\frac{O}{I} = \frac{50}{1--5} = \frac{50}{6} = 8.333$ i.e. O (= Z = Y) = $\frac{50}{6} * I$

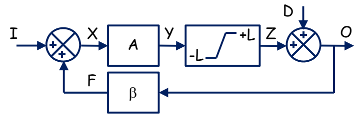
So when just at limit, Y = Z = O = 25, $\frac{25}{I} = \frac{50}{6}$ or I = 3

What happens when pass Limit?

A = 50

β = -0.1

L = 25



At limit

I = 3, O = 25

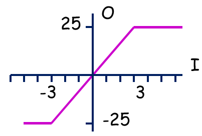
If I increases, X and Y increase, but Z and O stay at 25

Use same argument for limit -25;

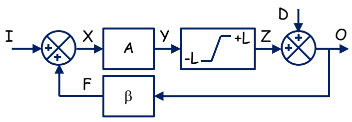
or say 'by symmetry' when I = -3, O = -25,

if I more -ve O stay -25

Hence (non linear) graph of OvI



General Case



In general, find I when Y = Z = O, ie limit block TF = 1

Then $\frac{O}{I} = \frac{L}{I} = \frac{A}{1-A\beta}$ so $I = L\frac{1-A\beta}{A}$

So, two limit points are $I = +L\frac{1-A\beta}{A}$ & $-L\frac{1-A\beta}{A}$

For our example, limit at $\pm 25\frac{1--50*0.1}{50} = \pm 25\frac{6}{50} = \pm 3$

Another example

I just limit at $+L\frac{1-A\beta}{A}$ & $-L\frac{1-A\beta}{A}$

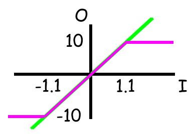
Suppose A = 100; β = -0.1 +L = 10 and -L = -10

System linear when -10 ≤ Y ≤ 10, then

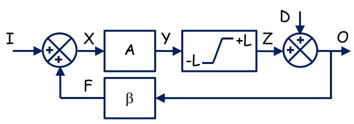
$$\frac{O}{I} = \frac{A}{1 - A\beta} = \frac{100}{11} \approx 9.09$$

Values of I where just limit are

$\pm 10 * \frac{1--10}{100} = 1.1$



Lecture 7 In Class Exercise



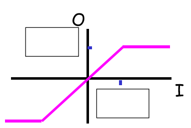
A = 1000

β = -1/5

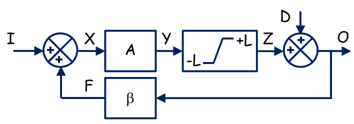
+L = 20

-L = -20

Calc O & I at L = 20 to label the graph



O vs D : a different response



Strategy, find D when just limit: Y = Z = ^{+}L: when $\frac{O}{D} = \frac{1}{1 - A\beta}$

Then $^{+}L + D = O$; $D = (1 - A\beta)*O$ so $^{+}L + O - A\beta O = O$

Hence $^{+}L = A\beta O$ or $O = \frac{^{+}L}{A\beta}$: $D = (1 - A\beta)\frac{^{+}L}{A\beta}$

Do similarly for when Y = Z = -L : or use symmetry

So, when A = 100; β = -0.1

If $-10 \le Y \le 10$, response as usual: $\frac{O}{D} = \frac{1}{1 - A\beta} = \frac{1}{11}$

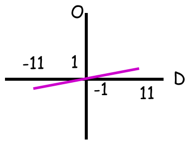
When just about to limit (remember $A\beta$ = 100*-0.1 = -10)

$O = \frac{+L}{A\beta} = \frac{10}{-10} = -1$; $D = (1-A\beta) * -1 = -11$;

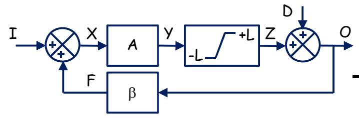
By symmetry, Y = Z = -10 when D = 11;

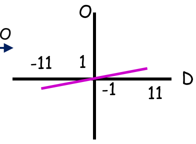
Hence, when $-11 \le D \le 11$, effects of D on O reduced by feedback.

But what happens when D > 11?



So, when A = 100; β = -0.1





Suppose D = 11 and O = 1

If D increased by 1, as Z stays at L, O up by 1

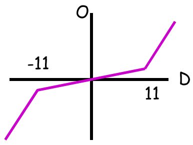
Similarly if D = -11 and O = -1

If D ↓ 1 to -12, O = -12 + 10 = -2. ie O ↓ 1

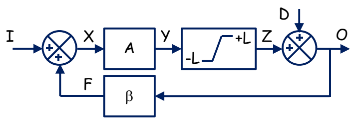
So when limiting, O/D now 1

In effect no feedback,

no reduction of effect of D on O

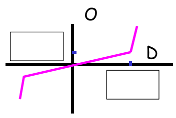


Lecture 7 In Class Exercise

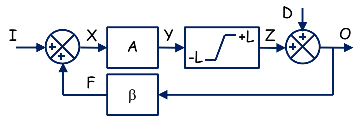


A = 1000
β = -1/5
+L = 20
-L = -20

Calc O & D at L = -20 to label the graph



Generalising



Found O/I if D = 0 and O/D if I = 0

What if I or D a non zero constant?

Remember, when linear, $O = \frac{A}{1-A\beta} I + \frac{1}{1-A\beta} D$

Hence, when Y = Z = L, $O = L + D = \frac{A}{1-A\beta} I + \frac{1}{1-A\beta} D$

So $(L + D)(1-A\beta) = AI + D$

Or $L + D - A\beta L - A\beta D = AI + D$

Or $L(1 - A\beta) - A\beta D = AI$

And so ..

So at limiting point, when O = L + D, $L(1 - A\beta) - A\beta D = AI$

For $\frac{O}{I}$, D constant, so $I = \frac{L(1-A\beta)}{A} - \beta D$

Hence limits are at

$O = L+D,\ I = \frac{L(1-A\beta)}{A} - \beta D$

$O = -L+D,\ I = \frac{-L(1-A\beta)}{A} - \beta D$

For $\frac{O}{D}$, $D = \frac{L(1-A\beta)}{A\beta} - \frac{I}{\beta} = \frac{L}{A\beta} - L - \frac{I}{\beta}$; so $O = D+L = \frac{L}{A\beta} - \frac{I}{\beta}$

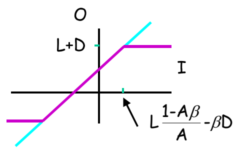
Hence limits are at

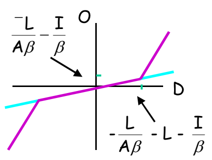
$O = \frac{L}{A\beta} - \frac{I}{\beta}$; D = O + L

$O = \frac{-L}{A\beta} - \frac{I}{\beta}$; D = O - L

Figures

Effect is to shift the graphs so not symmetrical about 0,0





$O = L+D,\ I = \frac{L(1-A\beta)}{A} - \beta D$ $\quad$ $O = \frac{L}{A\beta} - \frac{I}{\beta}$; D = O + L

$O = -L+D,\ I = \frac{-L(1-A\beta)}{A} - \beta D$ $\quad$ $O = \frac{-L}{A\beta} - \frac{I}{\beta}$; D = O - L

Limits and Hole in the Ozone Layer

The ozone layer is a feedback system

Must be (according to Gaia) so correct amount of u.v. getting to Earth's surface:

Too much u.v → cancer; too little → rickets

If ozone layer too thin, u.v. gets through : finds oxygen; turns it to ozone, thickens ozone layer: feedback!

Worked til too much CFC – which destroys ozone

CFCs are disturbances, and normally feedback reduces effects of CFCs

But when too much, system not cope as then has no feedback to reduce any extra disturbance

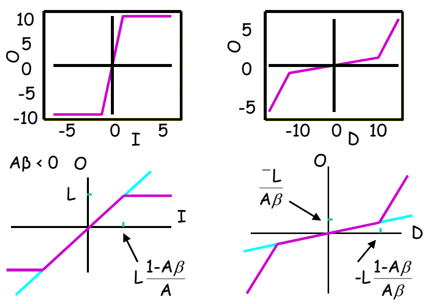
Limits Summary – MatLab code

Code below is a function to plot responses O/I or O/D (when DorI=0)
Each finds 'limit' points and then generates arrays of O and I or D

```
function fblims (A, b, lim, whatplot);   % plot limit system
OmAb = 1 - A*b;                                  % 1 minus A * b
Ilim = lim * OmAb / A;      % value of I when just limiting
if whatplot == 0         % -ve loop gain – plot O/I
  plot ([-Ilim-5, -Ilim, Ilim, Ilim+5], [-lim, -lim, lim, lim]);
else                     % -ve loop, plot O/D
  Olim = lim  / (A*b);   % O when just limits
  Dlim = Olim * OmAb;          % D when just limits
  plot([-Dlim-5, -Dlim, Dlim, Dlim+5], [-Olim-5, -Olim, Olim, Olim + 5]);
end
```


Example Output + General Summary

Graphs A = 100, β = ± 0.1, lim = 10



Summary

We have seen effect of limits on feedback systems

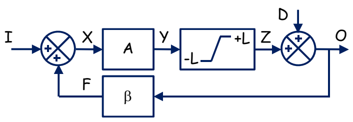
- We have considered examples when loop gain –ve
- We get different responses for O/I and O/D
- But we find using same strategy –
  - Find the values of O and (I or D) where just limit

Next week

- We consider what happens when loop gain +ve
- And see how, with an integrator, we can make square waves

8 : More Limits in Feedback Systems

Last week we introduced limits into feedback Systems



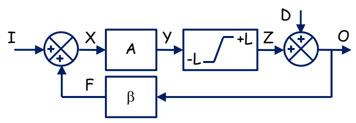
Where –L <= Y <= L, limit block has gain unity : Z = Y

But if Y > L, Z is L and if Y <-L, Z = -L

We considered the case where Aβ < 0

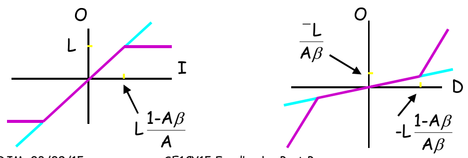
And determined O/I and O/ D separately

Limits in Feedback Systems

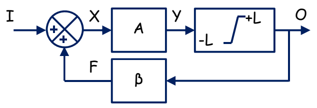


Strategy : find O and I or D, where just limit (Y = Z = L)

And then consider what happens to O when I or D exceeds these



Limits when have Positive Loop Gain



This week we consider what happens when Aβ > 0

Strategy : again find O and I when just limit (Y = Z = L)

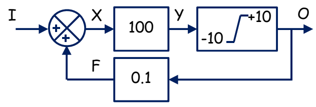
The response now is different

Interestingly we cant always find what O is just knowing I

Later we consider how with an integrator get square wave.

Let's start with an example …

Limits when have Positive Loop Gain



Suppose O = 10 and I = 1; F = 1, X = 2, Y = 200, so O = 10

If I reduced to -0.9: X = 0.1, Y = 10, O = 10 No change.

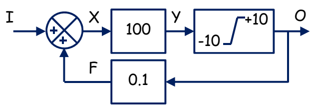
If I now -0.91: X = -0.91+1 = 0.09, Y = 9, O = 9

Then X = -0.01, Y = -1, O = -1

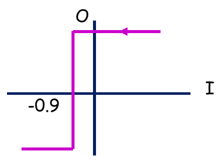
And then X = -1.01, Y = -101, O = -10

Very rapidly, O flipped +10 to -10 when I passed -0.9.

Limits and Positive Loop Gain



This change from +10 to -10 is represented graphically as



If I reduced now no change.

But if I increased to 0

X = -1, O = -10 still

Even if I upped to +0.9:

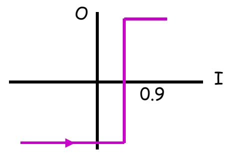
X = -0.1, O = -10, still, but..

How to Flip Back

If, however, I upped to 0.91,

X = I + O/10 = -0.09 O = 100 * X = -9

Then X = 0.01, O = 1; Then X = 1.01, Y = 101, O = 10

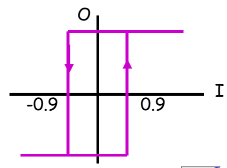


Thus, when I exceeds 0.9, O flips back to + 10

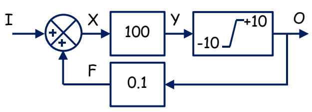
O stays at 10 until I again < -0.9.

O depends on I and on O!

Hysteresis: figure to right



How To Find I Where O Flips



Again find I when system just limits,

ie Y = O = 10

Limit block = 1

$\frac{O}{I} = \frac{100}{1\text{-}0.1*100} = -\frac{100}{9}$; or $I = -\frac{9}{100} * O = -\frac{9}{100} * 10 = -0.9$

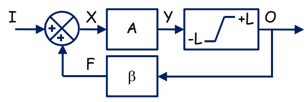
In general, flip when O = L, so $\frac{L}{I} = \frac{A}{1\text{-}A\beta}$ or $I = L * \frac{1\text{-}A\beta}{A}$

If I ≥ -0.9, O stays at 10, but if I < -0.9, O will flip to -10

By symmetry, I must exceed +0.9 for O to flip to +10

Could be found by same analysis.

Lecture 8 In Class Exercise



Suppose A = 100, β = 0.2, +L = 50, -L = 50.

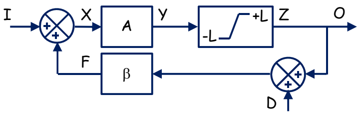
Initially, I = 0 and O = -50.

a) What must happen to I to make O change to +50

b) What must then occur to I for O to return to -50?

Now Consider Adding 'disturbance'

But Output of Limiter is System Output, D added in feedback



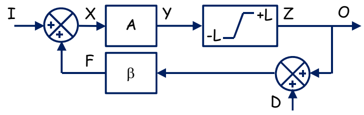
Again analyse by finding just limit point, when limit block = 1

Use Forward/1-Loop, now for O/I and O/D

$\frac{O}{I} = \frac{A}{1-A\beta}$ as usual, but $\frac{O}{D} = \frac{\beta A}{1-A\beta}$ as forward is βA

So $O = \frac{A}{1-A\beta}I + \frac{A\beta}{1-A\beta}D$

Finding 'flip' point



$O = \frac{A}{1-A\beta}I + \frac{A\beta}{1-A\beta}D$

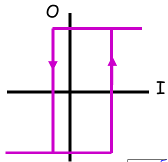
So, when O = Y = L

$L = \frac{A}{1-A\beta}I + \frac{A\beta}{1-A\beta}D$ or $L(1-A\beta) = AI + A\beta D$

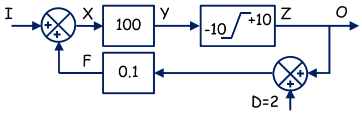
So $I = L\frac{(1-A\beta)}{A} - \beta D$

When O = Y = -L $I = -L\frac{(1-A\beta)}{A} - \beta D$

'flip' values not symmetrical about 0 ..



Lecture 8 – In Class Exercise



Calculate the values of I where Y = +10 and -10 and so plot OvI

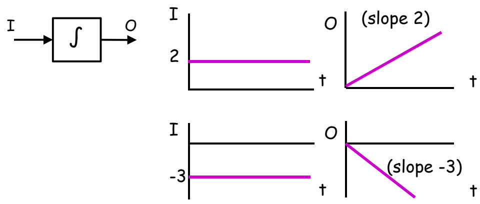
Adding Feedback – for Square Wave

We will add feedback to this … and an integrator …

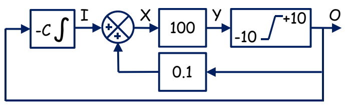
Let's consider integrators … eg pouring liquid into a container

If do so at constant rate, amount in container rises constantly

If drink at constant rate, amount decreases constantly



Application: Square Wave Generator



'hysteresis' loop input,

$I = -C * \int O$

I changes at rate -CO

Hysteresis loop : O becomes 10 when I>0.9, -10 when I<-0.9

Suppose O = 10 initially and I = 0.9, and C > 0

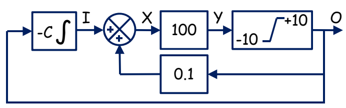
-C*O < 0, so I will decrease …

When I reaches -0.9, O becomes -10

Now -C*O > 0, so I will increase

When I reaches +0.9, O becomes +10 …

Application: Square Wave Generator



At t = 0,

I = +Flip,

O = +Lim

At time t, I = Flip - C*Lim*t

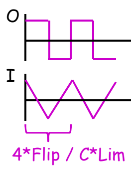
So at t = 2*Flip / C*Lim, I = -Flip

Then O := -Lim

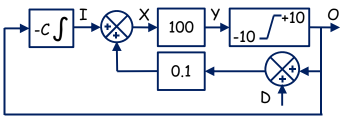
After a further time of 2*Flip / C*Lim

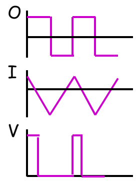
I = +Flip, so O := +Lim

Square wave +/-Lim, period 4*Flip / C*Lim

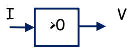


If add 'D' to loop





Period unchanged, but I will vary between different values.
But if pass I to comparator

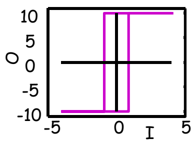


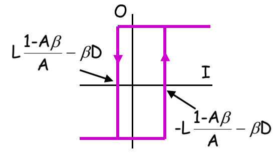
Get uneven mark space ratio square wave – used in robots, as we shall see later …

MatLab code and Summary

```
function fblims (A, b, lim, D);   % plot limit system
OmAb = 1 - A*b;                   % 1 minus A * b
Ilim = lim * OmAb / A;     % value of I when just limiting
if A*b > 0               % +ve loop gain – plot hysteresis
   plot([-Ilim-5, Ilim-b*D, Ilim-b*D, Ilim+5, …
        -Ilim-b*D, -Ilim-b*D, -Ilim-5], …
        [-lim, -lim, lim, lim, lim, -lim, -lim]);
```

If A = 100,
$\beta = \pm 0.1$,
lim = 10,
D = 0





Summary

This week we have looked again at feedback systems and limits

Here loop gain is positive, and we found 'hysteresis' effect

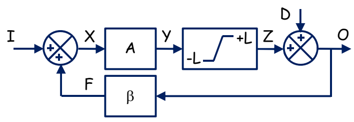
Adding 'disturbance' means this not symmetrical about 0

We have seen effect of a constant into an integrator

And how an integrator in loop around limited system can be used to generate a square wave.

Before next week, try following exercise on blackboard

Lecture 8 After Class Exercise



Suppose A = 100, β = 0.2, +L = 50, -L = 50.

Initially, I = 0 and O = -50.

a) What must happen to I to make O change to +50

b) What must then occur to I for O to return to -50?

9 : Electronics and Feedback

We have seen and analysed feedback control systems

Where the aim is that output = input

And a more general feedback system

Where the aim is often output = input * value

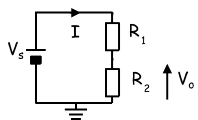
In this lecture we consider electronic systems

Which are also often feedback systems

We have previously shown how a potential divider is feedback

We will build on that…

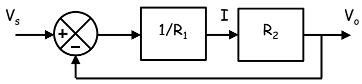
Potential Divider is Feedback System!



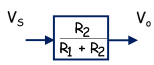
Kirchhoff : $V_S = I * R_1 + V_O$, so

$I = \frac{V_S - V_O}{R_1}$

$V_o = R_2 * I$, so



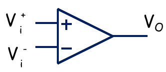
$$\frac{V_O}{V_S} = \frac{\frac{1}{R_1}R_2}{1 - -\frac{1}{R_1}R_2} = \frac{R_2}{R_2 + R_1}$$



We will use this block later

Now another component

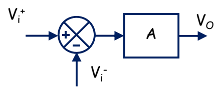
Operational Amplifier



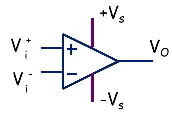
$V_o = A * (V_i^+ - V_i^-)$

A very big, ~10^5,

We can consider this in block diagram terms :



Needs a power supply



V_s typically 10 or 15V

Limits V_o

As A is very big and V_O is limited (say between -10 and +10V)

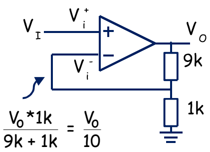
Then $V_i^+ - V_i^- < 10/10^5 = 10^{-4}$ ie very small

So we can approximately say $V_i^+ = V_i^-$

In fact, we can use feedback to achieve this.

Example

There are various ways of using feedback – a simple one



$$\frac{V_O*1k}{9k + 1k} = \frac{V_O}{10}$$

The fedback signal is from a potential divider ... so

$$V_i^- = \frac{V_O*1k}{9k + 1k} = \frac{V_O}{10}$$

But, as we have said, $V_i^+ = V_i^-$

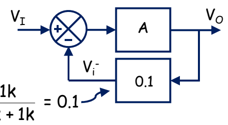
So $V_I = V_O/10$,

or $V_O = 10 * V_I$

Analysis with Block Diagrams

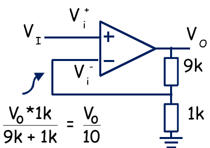
Assume A very big, 10^5

Block Diagram for system



$\beta = \frac{1k}{9k + 1k} = 0.1$

(from earlier)



$$\frac{V_O*1k}{9k + 1k} = \frac{V_O}{10}$$

High loop gain approx

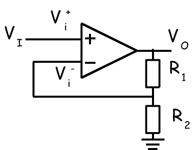
$$\frac{V_O}{V_I} = \frac{A}{1 - -A*0.1}$$

$$\approx \frac{A}{A*0.1} = 10$$

In fact $\frac{V_O}{V_I} = \frac{10^5}{1+10^4} = 9.999$

Very close!

Lecture 9 - In Class Exercise



Find V_o if V_I = 1V, OpAmp gain A = 1000, R_1 = 80kΩ and R_2 = 20kΩ.

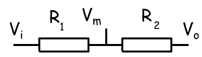
Note, find the exact value, and compare with the approximate one.

By how much does V_o change, if A increases by 10% to 1100?

Does the system have negative feedback?

Now, for another circuit

As preparation for another op-amp circuit consider this…



Want V_m in terms of V_i and V_o.

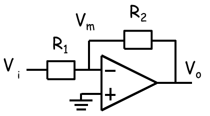
Use Superposition, find V_m when V_o = 0 and when V_i = 0 ..

If $V_O = 0$, $V_m = \frac{R_2}{R_1 + R_2} V_i$ If $V_i = 0$, $V_m = \frac{R_1}{R_1 + R_2} V_O$

Hence $V_m = \frac{R_2}{R_1 + R_2} V_i + \frac{R_1}{R_1 + R_2} V_O$

Virtual Earth Amplifier

One input connected to earth (0V), so other is virtually 0V …



Here the other is mid point of two resistors

$$V_m = \frac{R_2}{R_1+R_2} V_i + \frac{R_1}{R_1+R_2} V_o$$

$$V_m \approx 0, \text{ so } \frac{R_2}{R_1+R_2} V_i = -\frac{R_1}{R_1+R_2} V_o$$

$$\frac{V_o}{V_i} \approx -\frac{R_2}{R_1}$$

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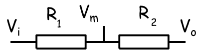
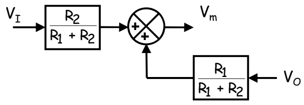
Block Diagram of this

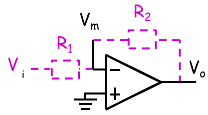
First for the two resistors

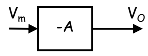


Hence $V_m = \frac{R_2}{R_1+R_2} V_i + \frac{R_1}{R_1+R_2} V_o$

Block Diagram







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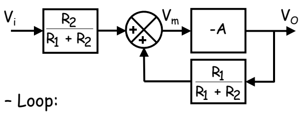
Block Diagram Analysis

Combining these we get



Forward / 1 – Loop:

$$\frac{V_o}{V_i} = \frac{\frac{R_2}{R_1+R_2} * -A}{1 - -A\frac{R_1}{R_1+R_2}}$$

$$= \frac{-AR_2}{R_1 + R_2 + AR_1} \approx -\frac{R_2}{R_1}$$

A big, so $AR_1 \gg R_1 + R_2$

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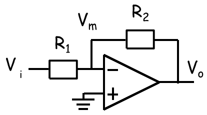
With Values

Suppose R_1 = 1kΩ and R_2 = 9 kΩ

By approximate analysis

$$\frac{V_o}{V_i} \approx -\frac{R_2}{R_1} = -9$$



By full analysis, again assume $A = 10^5$:

$$\frac{V_o}{V_i} = \frac{\frac{R_2}{R_1+R_2} * -A}{1 - -A\frac{R_1}{R_1+R_2}} = \frac{\frac{9}{10} * -10^5}{1 + 10^5 \frac{1}{10}} = \frac{9}{10} * \frac{-10^5}{10001} = -8.999$$

High loop gain approx valid

http://www.reading.ac.uk/~shsmchlr/jscyber/tfstatic.html

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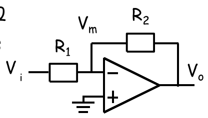
Lecture 9 In Class Exercise

A = 1000, R_1 = 20kΩ and R_2 = 80kΩ

Find Vo/Vi by approximate analysis

And then confirm by full…



Approximately $\frac{V_o}{V_i} \approx -\frac{R_2}{R_1} = -\frac{80k}{20k} = -4$

By full analysis:

$$\frac{V_o}{V_i} = \frac{\frac{R_2}{R_1+R_2} * -A}{1 - -A\frac{R_1}{R_1+R_2}} = \frac{\frac{80k}{20K+80K} * -1000}{1 + 1000\frac{20k}{20K+80K}} = -\frac{0.8*1000}{1+0.2*1000} = -\frac{800}{201} = -3.98$$

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On Limits

As we have said, op-amps have a power supply $\pm V_s$

In practice, Vo is limited a bit more than to $\pm V_s$.

We define two more voltages :

$+V_{sat}$ is the most positive output voltage, saturation voltage

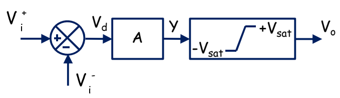
$-V_{sat}$ is the most negative output voltage, saturation voltage

$-V_{sat} < V_o < +V_{sat}$

Defining $V_d = V_i^+ - V_i^-$

Hence

$$-\frac{V_{sat}}{A} < V_d < +\frac{V_{sat}}{A}$$



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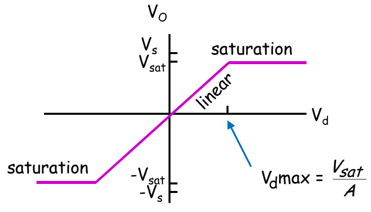
Graph of V_o vs V_d



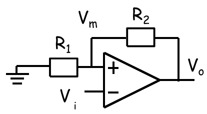
Note: linear and non linear mode of operation for the op-amp.

Observe that, V_{sat} value is close but not equal to the V_s value.

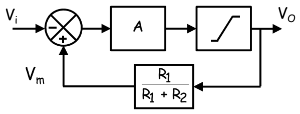
So far we have used op amp circuits in linear mode, but …

Using Op-Amp with Limits

Consider this … which is not the same as virtual earth



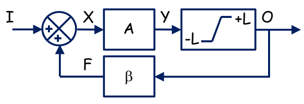
Block diagram is thus

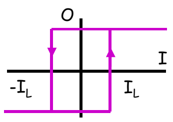


Which (other than the – sign) is what we met last week ..

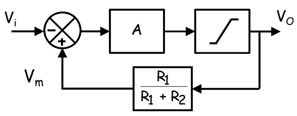
And so

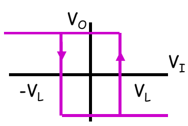
Last week ..





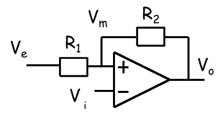
Now with the - sign



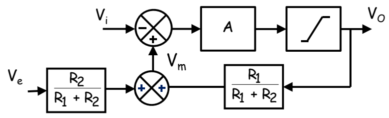


Using Op-Amp with Limits

And by extension



Block diagram is thus



Which shifts the hysteresis graph horizontally

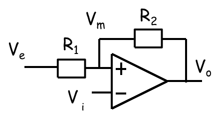
Summary

This week we have looked at electronic feedback systems

- Reminding us that the potential divider is feedback
- Then introducing the operational amplifier
- And how it can be used with feedback
  - As a non-inverting amplifier
  - And as an inverting virtual earth amplifier
- In addition, we have seen that the op-amp has limits
  - And so can generate hysteresis effects

Next week, we introduce dynamic systems …

Lecture 9 – After Class Exercise



Suppose R_1 = 20K, R_2 = 80K, V_{sat} = 10V and amp gain A = 1000

When V_e = 0, find where system linear and so plot V_o v V_i

Then, suppose V_e = 1V, and so find the limits

$$\frac{V_O}{-V_I} = \frac{1000}{1 - 0.2*1000} = \frac{1000}{-199};\ \text{so}\ \frac{10}{-V_I} = \frac{1000}{-199}\ \text{or}\ V_I = 1.99\ \text{(and also -1.99)}$$

$$V_O = -V_I \frac{1000}{-199} + V_e * \frac{0.8*1000}{-199};\ -199*10 = -V_I*1000 + 800\ \text{so}\ V_I = 2.79$$

and for -10 limit, -199*-10 = $-1000V_I$ + 800, or V_I = -1.19

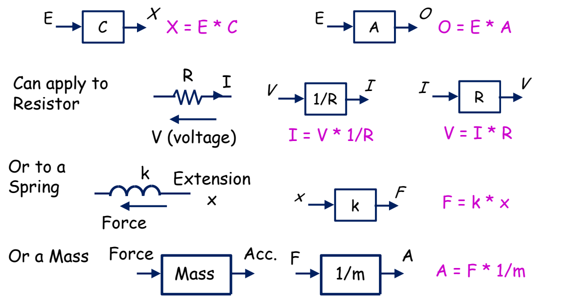
10 – Introducing Dynamic Systems

We have mainly looked at static systems
- Here the system components are fixed
- Where we work out the output given the input
  - (and past output in positive loop gain limited systems)
- These include control systems and Aβ systems
- We have also shown how these can be implemented in electronics
- We have found transfer functions Output = Input * constant

For the rest of the feedback course we consider dynamic systems
- Here we work out how the output gets to its value
- We will introduce some concepts in this lecture

Blocks we have met

We have met components where its output = its input * constant



But

In some cases we need a more sophisticated block

Remember when manually controlling the robot
- If robot travelling too slow, increase speed
- If travelling too fast, decrease speed

These mean that the controller block cant be
- output = input * value

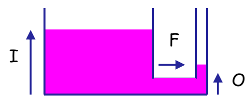
Instead it must be
- output = output + input * value

This, we said earlier was the process of integration.

Let's develop the associated ideas...

Two tanks connected by pipe

This is first of two examples of systems with inherent feedback and blocks of output = input * constant are not sufficient



Pipe has resistance - sets flow through it

Tanks have capacity - sets rate height of liquid changes

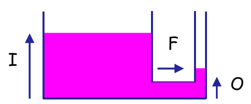
Liquid flows due to difference in pressure at ends of pipe

$$\text{Pressure} = \frac{\text{Weight}}{\text{Area}} = \frac{\text{Density*g*Volume}}{\text{Area}}$$

$$= \frac{\text{Density * g*Height * Area}}{\text{Area}} = \text{constant * Height}$$

So difference in Height causes Flow through pipe

Block Diagram of Tanks

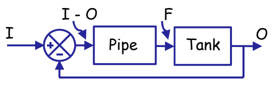


Flow F = (I-O) / Resistance

F is input * constant

Flow F means O rises

Hence, block diagram



System reaches final state 'steady state', when signals constant

This will be when F = 0, which is when O = I

NB: summer does I – O: so signals I and O same type (units m)

Flow F is volume moving at rate : units m^3 / s

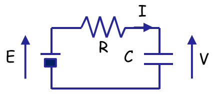
Analogous Systems

Often same model can apply to two different system types

Water system
- water flows thru pipe as pressure difference across pipe
- pipe has resistance (affected by its size): restricts flow
- water flows into tank; means height of water increases
- speed at which height rises affected by tank's capacity

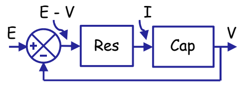
Electronic system
- current flows thru resistor, as voltage difference across it
- resistor has resistance – which resists current flow
- current flows into capacitor; so voltage across it increases
- speed of voltage rises affected by capacitor's capacitance

Resistor-Capacitor System



Voltage input, E, from battery.
Output V, across capacitor

Voltage across Resistor, E – V, determines I
I into capacitor causes V to increase



NB I = $\frac{E-V}{R}$

so Res block is $\frac{1}{R}$

Final, Steady value, when V = E, then I = 0
E and V measured in volts V, I measured in amps A

*When Output NOT Input * Constant*

For water tank, the height of water increases when water flows in
Height = Height + amount due to flow F

For a capacitor, current flowing in causes voltage out to increase
V = V + amount due to I

For motor, net Force means motor accelerates ie velocity changes
Velocity = Velocity + amount due to acceleration
(we hid that detail in robot example)

Mathematically, 'integration' does this sort of operation
Output = Output + amount * Input

Hence blocks can include Integrators (output constant if input 0)

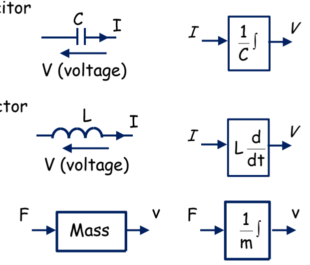
Can also have blocks which reflect change : differentiation

Examples

Capacitor

Inductor

Mass

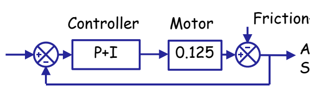


V = 1/C * Integral I

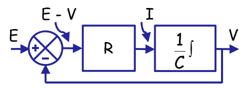
V = L * Differential I

v = 1/m * Integral (F)

On Integrators and Final Value



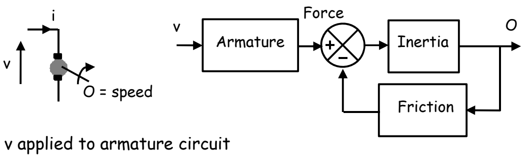
In the robot example with P+I control, Actual = Desired
Because, Actual constant if output of controller constant
Which means input to controller is 0, ie Desired – Actual = 0



We argued, V finally becomes E, which (as Cap is integrator) requires its input I to be 0 – which happens when E-V = 0

Motor

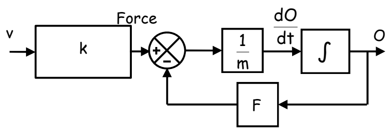
Let's look at motors a bit more



v applied to armature circuit
→ current i
→ force to make motor move (= k * v)
→ motor moves, output velocity O
→ friction (which depends on velocity) opposes motion

Consider the Inertia

Net Force = that due to Current – that due to Friction (F * O)
Net Force = mass * acceleration (Newton's 2nd Law)
Acceleration is change in (differential of), velocity
We want velocity, so we integrate for O



Steady velocity is when input to integrator is zero
Which is when v * k = F * O … So O = v * k / F

Rotational Movement

In the above example, the motor moves in straight line

Its position (units m) changes, it moves with a given velocity (m/s)

It has mass (kg), it accelerates (ms^{-2}), due to force (N)

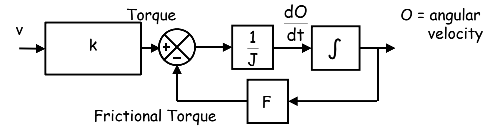
Can also have motor which rotates with equivalent concepts

Linear Motion	Units	Rotational	Units
Position	m	Angular Position	radians (rads)
Velocity	$m\ s^{-1}$	Angular Velocity	rads s^{-1}
Acceleration	$m\ s^{-2}$	Angular Acceleration	rads s^{-2}
Force	N	Torque	N m
Mass	kg	Moment of Inertia	kg m^2

Rotary Motion

Current generates input Torque = MoI * angular acceleration

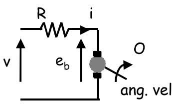
Angular Acceleration is differential of angular velocity

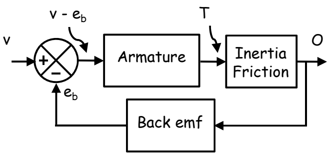


Steady angular velocity is when input to integrator is zero

Which is when v * k = F * O … So O = v * k / F

Permanent Magnet Motor





v applied to armature circuit: motor turns : e_b 'back emf' generated

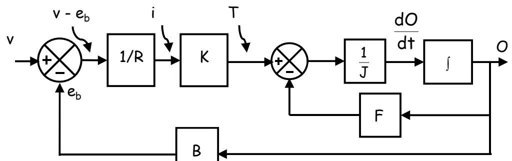
Difference between v and e_b → V across R → current, i

i → torque T (= K*i) → angular velocity O (note inertia/friction)

O → e_b (= B*O)

Slightly more complicated model ..

So Full Block Diagram is



Find constant angular velocity .. where $\frac{dO}{dt} = 0$

$T = (v - O*B) * \frac{1}{R} * K = F * O$ $\qquad v * \frac{K}{R} = F * O + O*B* \frac{K}{R}$

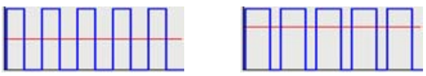
$v * K = O * (F * R + B * K)$ $\qquad O = v * \frac{K}{F * R + B * K}$

How we set motor speed by computer

Computer outputs only two values, high and low

So how get variable speeds?

Answer we use square waves (but with variable mark/space ratio)



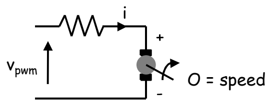
Signal changes rapidly ..

But average value constant – for given mark/space

Send to motor, which turns at the rate set by the average value.

Called Pulse Width Modulation

How do we set Direction



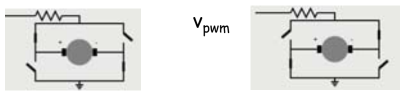
Connect PWM signal to motor

Average sets speed clockwise

Reverse wires for anticlockwise

Have switches to in effect do this … A bridge circuit

v_{pwm} $\qquad$ v_{pwm}



Two switches open, two closed …

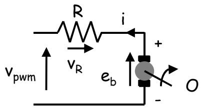
Sets whether v_{pwm} connected to + or to -

Measuring Speed

PWM sets nominal speed, but disturbances may slow iy

So somehow need to measure motor speed.

Can add extra hardware (eg encoders)... But PWM can help



Use back emf e_b proportional to speed

Note resistor in circuit

Kirchhoff: $V_{pwm} + v_R = e_b$

At end of PWM cycle $v_{pwm} = 0$, when $v_R = e_b$

So computer generating PWM measures v_R then

And so determines the speed

See http://www.reading.ac.uk/~shsmchlr/jscyber/demoPWM.html

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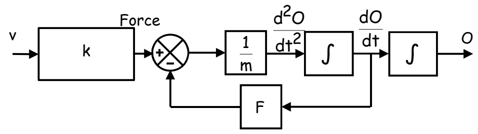
What if output is motor position?

Position is integral of velocity, so ...



Some similarities with a water clock ...

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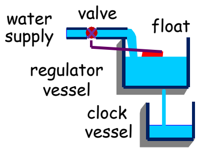
Water Clock has integrators !

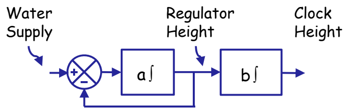


Water flows if float valve drops

Height increases – so integrator

Water flows into clock vessel

It too is an integrator



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Summary

This lecture has been about introducing dynamic systems

We now have the concept of blocks with integrators

- And have seen this applies to electronics (capacitors)
- As well as motors and even water clocks

We have also worked out final values –

- Where input to integrators are 0

What we need to do is to work out how systems reach the final value

Where we can again use transfer functions and Forward/1-Loop

Which we will do next term ...

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