SE1CY15 Cybernetics and Circuits
Feedback – Part C
Prof Richard Mitchell

In the third quarter of the course the topics are

- Dynamic Feedback Systems
- Frequency Response
- Use of MatLab
- Introduction to time domain analysis

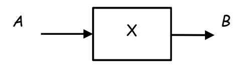
These will continue to be assessed by computer based labs

The topics build on last terms lectures

In this lecture we start by reminding us of these topics.

11 : Systems - a reminder

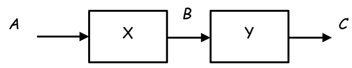
We analyse systems with inputs and outputs



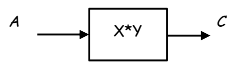
b = a*X (or X *a) or $\frac{b}{a} = X$

X is Transfer Function (TF)

Two blocks in series



Combine, for single TF

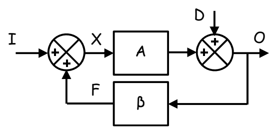


Feedback Systems

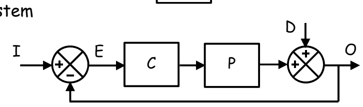
We have considered two forms of feedback system

Have input I, output O, with disturbance D

General

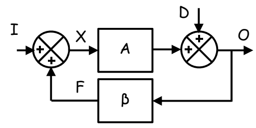


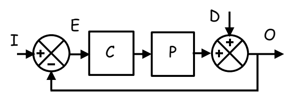
Control System



Analyse with 'forward over 1 minus loop' rule for overall TF

Using Forward over 1 minus Loop





Forward is TF input to O no loop; Loop is TF round loop

D = 0, Forward = A, Loop = Aβ $\quad \frac{O}{I} = \frac{A}{1 - A\beta}$

Forward = CP, Loop = -CP $\quad \frac{O}{I} = \frac{C*P}{1 + C*P}$

I = 0, Forward, D..O = 1, Loop = Aβ $\quad \frac{O}{D} = \frac{1}{1 - A\beta}$ $\quad \frac{O}{D} = \frac{1}{1 + C*P}$

Hence by Principle of Superposition $\quad O = \frac{A}{1 - A\beta} I + \frac{1}{1 - A\beta} D$ $\quad O = \frac{C*P}{1 + C*P} I + \frac{1}{1 + C*P} D$

High Loop Gain

It is important for loop gain to be high (-ve or +ve)

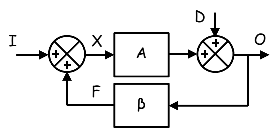
$$O = \frac{A}{1 - A\beta} I + \frac{1}{1 - A\beta} D \qquad O = \frac{C*P}{1 + C*P} * I + \frac{1}{1 + C*P} * D$$

Aβ big so 1 - Aβ ~ -Aβ $\qquad$ CP big so 1 + CP ~ CP

$$O \approx \frac{1}{-\beta} * I + 0 * D = -\frac{1}{\beta} * I \qquad O \approx \frac{CP}{CP} * I + \frac{1}{CP} * D = I$$

O set by I and β	O set by I
largely unaffected if A changes.	largely unaffected if P changes.
largely unaffected by D.	largely unaffected by D.

Lecture 11 In Class Exercise



Suppose

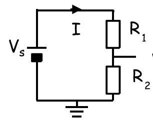
A = 990

β = -0.1

a) Find 1 minus Loop
b) Find O/I assuming D = 0
c) Find O/D assuming I = 0
d) Evaluate O if I = 10 and D = -5
e) Find O/I if A changed to 1000

Potential Divider

One of many electronic circuits that are feedback systems



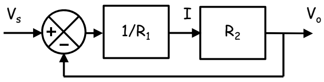
$V_o = \frac{R_2}{R_1 + R_2} V_s$

$I = \frac{V_S - V_O}{R_1}$

And $V_o = I * R_2$

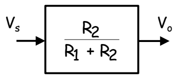
Block Diagram

Shows feedback!

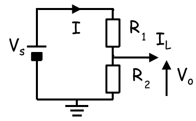


$\frac{V_O}{V_S} = \frac{\text{Forward}}{1 - \text{Loop}} = \frac{\frac{1}{R_1} R_2}{1 - -\frac{1}{R_1} R_2} = \frac{R_2}{R_2 + R_1}$

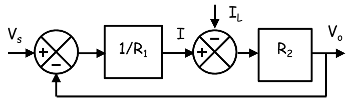
So can model as



Potential Divider with load:



Now $V_o = R_2 * (I - I_L)$, so



If $I_L = 0$, $\frac{V_O}{V_S} = \frac{\frac{1}{R_1} R_2}{1 - -\frac{1}{R_1} R_2} = \frac{R_2}{R_2 + R_1}$

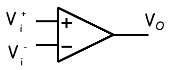
If $V_S = 0$, $\frac{V_O}{I_L} = \frac{-R_2}{1 - -\frac{1}{R_1} R_2} = -\frac{R_1 R_2}{R_2 + R_1}$

By superposition:

$V_O = \frac{R_2}{R_2 + R_1} V_S - \frac{R_1 R_2}{R_2 + R_1} I_L$

Operational Amplifier

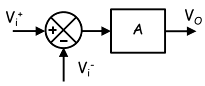
Also get feedback with operational amplifier (op amp) circuits



Two inputs and one output

$V_o = A * (V_i^+ - V_i^-)$

Model : summer + block with gain A



A very big, ~10^5, so if V_o say in range -10 to +10V

$V_i^+ - V_i^- = V_o/A \sim 0$: so $V_i^+ = V_i^-$

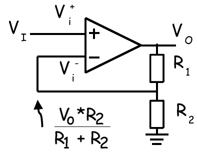
To achieve this, we put feedback round them .. Such as

Operational Amplifier + Pot Divider

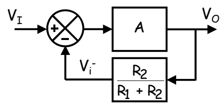
V_i^- set by potential divider

But $V_i^- = V_i$ approximately:

So $V_I = V_O \frac{R_2}{R_1 + R_2}$ or $\frac{V_O}{V_I} = \frac{R_1 + R_2}{R_2}$



Block Diagram for complete analysis



$\frac{V_O}{V_I} = \frac{A}{1 - -A \frac{R_2}{R_1 + R_2}}$

$\approx \frac{A}{A \frac{R_2}{R_1 + R_2}} = \frac{R_1 + R_2}{R_2}$

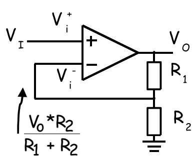
http://www.reading.ac.uk/~shsmchlr/javascript/transfunc.html

Put Some Values In

Suppose R_1 = 9 kΩ and R_2 = 1 kΩ

By approximate analysis

So $V_I = V_O \frac{1k}{9k + 1k}$ or $\frac{V_O}{V_I} = \frac{10k}{1k} = 10$



For full analysis, assuming A is 10^5

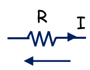
$\frac{V_O}{V_I} = \frac{10^5}{1 - -10^5 \frac{1k}{9k+1k}} = \frac{10^5}{1 + 10^5 \frac{1}{10}} = \frac{10^5}{1 + 10000} = 9.999 \approx 10$

Is Aβ system: β = -0.1 : O/I ~ -1/ β = 10 : |1-A β| = 10001 > 1

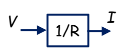
Blocks which Integrate

When we model a resistor, we use V = I * R or I = V/R

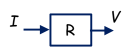
Output is a constant proportional to input



V (voltage)



I = V * 1/R

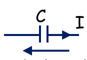


V = I * R

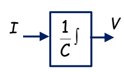
But, when I flows into a capacitor, the voltage V across it rises

V = V + amount due to I

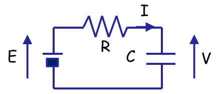
V = 1/C * Integral I



V (voltage)



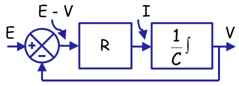
RC Circuit



Voltage input, E, from battery.

Output V, across capacitor

$I = \frac{E-V}{R}$ and $V = \frac{1}{C}\int I\,dt$



If V = 0 initially, it will then rise as I flows.

When will it stop rising? When input to integrator is 0.

That is when I = 0, which is when E-V = 0 or V = E

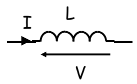
This is its STEADY STATE value

Differentiator

We note that a capacitor is modelled by an integrator

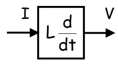
V = 1/C * Integral I

Another electronic component is an inductor



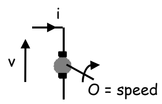
$V = L\frac{dI}{dt}$

So a block diagram is



Point to note, we may need to integrate or to differentiate.

Also need Integrators for Motors



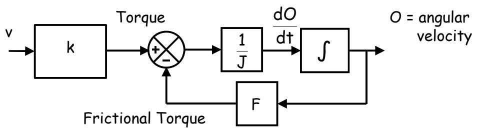
Voltage applied – means current flows

Generates torque making motor turn

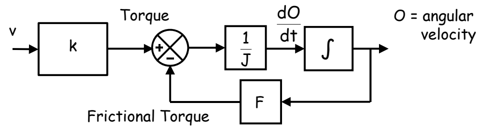
Frictional torque opposes motion

NetTorque = MoI * angular acceleration

Integrate this to get angular velocity



For Motor



For steady state:

$\frac{dO}{dt}$ = 0 means v*k - F*O = 0

Hence $O = \frac{v*k}{F}$

Two Key Questions

We have found the steady state value of the output

V is steady state when V = the constant E

Speed O is steady state when O = v*k/F constants

This is true if the system input (E or v) is constant (after t=0)

What if it isn't?

Also … if O = 0 at time 0,

How does O get to its steady value?

Summary

In this lecture have reminded ourselves of

Block Diagrams Feedback Systems : Forward / 1 – Loop

Importance of High Loop Gain

We have also looked at electronic circuits with feedback

The potential divider and op-amps

We have also considered blocks with integrators / differentiators

We can work out steady values for constant inputs

And posed two questions

Next week we address the first .. Assuming inputs are sinusoids

And start using complex numbers which actually make it easier …

12: Sinusoids and Feedback

Last week we reminded ourselves about feedback systems

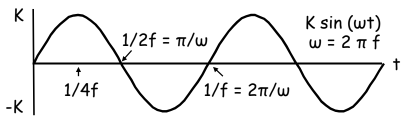
And looked at some electronic and motor systems

- We model these by simple blocks we combine
- Some blocks have the form output = input * value
- But some are integrators or differentiators.
- We worked out the steady state output if the input is a step

In this lecture we analyse systems where the input is a sinusoid

- Now we will see how blocks can process sinusoids
- And model integration/differentiation using $\sqrt{-1} = j$
- We will introduce in context of electronics, but applies elsewhere

When Signals are Sinusoids



f is frequency

ω is angular f

K is amplitude

ωt is an angle

Note that cos is a shifted sin : $\pm \cos(\omega t) = \sin(\omega t \pm \frac{\pi}{2})$

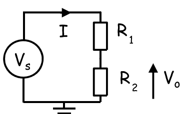
Also $p \sin(\omega t) + q \cos(\omega t) = \sqrt{p^2 + q^2} \sin(\omega t + \tan^{-1}\frac{q}{p})$

The angle a in $\sin(\omega t + a)$ is termed a phase shift

If one signal in system is a sinusoid, all others are sinusoids of same angular frequency with different amplitudes + may be phase shifted

Applies to all linear systems – found easily using complex numbers

Consider Pure Resistive System



Suppose R_1 = 1 kΩ and R_2 = 9 kΩ

Also, suppose V_s = 5 sin(7t)

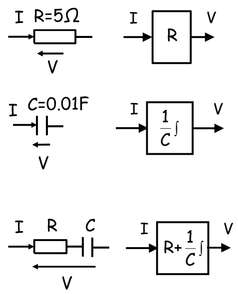
$$V_o = \frac{R_2}{R_1 + R_2} V_s = \frac{9}{10} 5 \sin(7t) = 4.5 \sin(7t)$$

$$I = \frac{V_s}{R_1 + R_2} = \frac{5 \sin(7t)}{10k} = 0.0005 \sin(7t)$$

All 3 signals, same frequency, different amplitude, no phase shift

http://www.reading.ac.uk/~shsmchlr/javascript/SinAndRC.html

Sinusoidal Currents into R and/or C



I = sin(40t)

V = 5 sin(40t)

$V = \frac{1}{0.01} \int \sin(40t)\, dt$

$= -\frac{1}{0.01 * 40} \cos(40t)$

= -2.5 cos(40t)

$= 2.5 \sin(40t - \frac{\pi}{2})$

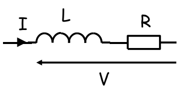
V = 5 sin(40t) - 2.5 cos(40t)

$= \sqrt{5^2 + 2.5^2} \sin(40t + \tan^{-1}(\frac{-2.5}{5}))$

= 5.59 sin(40t - 0.464)

V = V across R + V across C

In Class Exercise



Suppose I = sin(40t), L = 0.2H and R = 6Ω

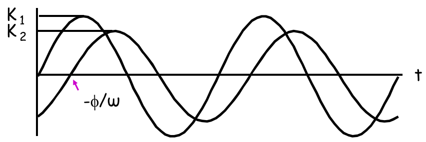
Find $V = I R + L \frac{dI}{dt}$ as phase shifted sin

These illustrate a Key General Point

For **any** linear system, *under steady state conditions*,

If input is $K_1 \sin(\omega t)$, output is $K_2 \sin(\omega t + \phi)$

Sinusoid, same ang freq, diff amplitude and phase shifted

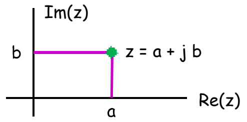


For block diagram analysis, need blocks which can both change amplitude and do a phase shift (ie angle shift) ...

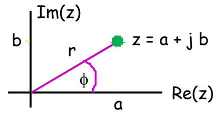
We need numbers which have size and angle ...

Complex Numbers Give Size and Angle

$z = a + j * b$ a is real part; b is imaginary part; $j = \sqrt{-1}$



Plot on 2D graph, the Argand plane



Same point also defined by distance from 0,0 and angle from real axis

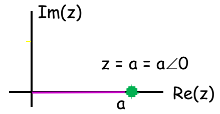
Distance r is modulus

Angle ϕ is argument

$|z| = r = \sqrt{a^2 + b^2}$

$\angle z = \phi = \tan^{-1}\left(\frac{b}{a}\right)$

Two Points to Note



A 'normal' number is a special case of a complex number

A point on 'real' axis

Value = distance from 0, angle 0

For systems often have $z = \frac{z_1}{z_2}$

modulus and argument easy : $|z| = \frac{|z_1|}{|z_2|}$ and $\angle z = \angle z_1 - \angle z_2$

$z = \frac{a + j\,b}{c + j\,d}$ $|z| = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$ $\angle z = \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{d}{c}\right)$

Integration and Complex Numbers

Consider $\frac{1}{j\omega}$: $\left|\frac{1}{j\omega}\right| = \frac{\sqrt{1^2 + 0^2}}{\sqrt{0^2 + \omega^2}} = \frac{1}{\omega}$

$\angle\frac{1}{j\omega} = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{1}{0}\right) = -\tan^{-1}(\infty) = -\frac{\pi}{2}$

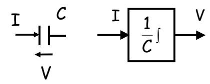
Has size $\frac{1}{\omega}$ and angle $-\frac{\pi}{2}$

But $\int \sin(\omega t) = -\frac{1}{\omega}\cos(\omega t) = \frac{1}{\omega}\sin(\omega t - \frac{\pi}{2})$

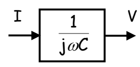
Change of size by $\frac{1}{\omega}$ and change angle by $-\frac{\pi}{2}$

So $\frac{1}{j\omega} \equiv \int$ Similarly, $j\omega \equiv \frac{d}{dt}$

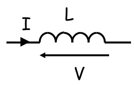
So Model for Capacitor and Inductor



$V = \frac{1}{C}\int I\,dt$ model by $V = I * \frac{1}{j\omega C}$

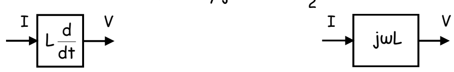


Consistent with complex impedance $Z = \frac{1}{j\omega C}$

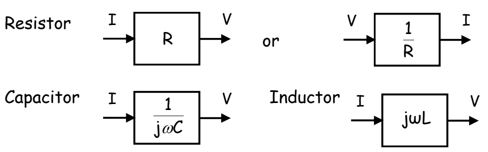


Inductor: $V = L\frac{dI}{dt} = \omega L\cos(\omega t) = \omega L\sin(\omega t + \frac{\pi}{2})$

Model by $j\omega L = \omega L\angle\frac{\pi}{2}$ $Z = j\omega L$

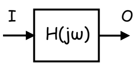


Hence for Block Diagrams



And then

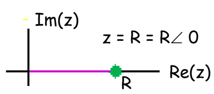
For any System, where I = sin(ωt), we model it by a complex transfer function, H(jω), and readily determine O …



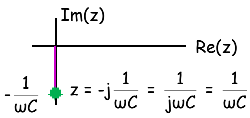
$O = |H(j\omega)|\ \sin(\omega t + \angle H(j\omega))$

Argand Plot for R, C and R + C

If I is sin(ωt), find V across component(s)

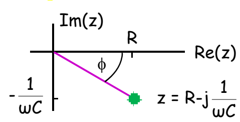


$V = R\sin(\omega t)$



$z = -j\frac{1}{\omega C} = \frac{1}{j\omega C} = \frac{1}{\omega C}\angle-\frac{\pi}{2}$

$V = \frac{1}{\omega C}\sin(\omega t - \frac{\pi}{2})$



$z = R - j\frac{1}{\omega C}$

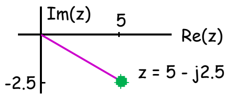
$= |z|\angle-\phi$

$V = |z|\ \sin(\omega t + \phi)$

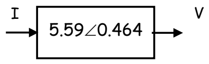
$= \frac{1}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}\sin\left(\omega t - \tan^{-1}\frac{1}{\omega CR}\right)$

With Some Values

Earlier example, $I = \sin(40t)$, $R = 5\Omega$, $C = 0.01F$, $1/\omega C = 2.5$



$$z = 5 - j2.5 = \sqrt{5^2+2.5^2}\ \angle\tan^{-1}\frac{-2.5}{5} = 5.59\angle -0.464$$



So $V = 5.59 \sin(40t - 0.464)$

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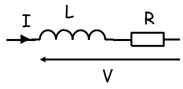
In Lecture Exercise



$I = \sin(40t)$, $L = 0.2H$ and $R = 6\Omega$

Do Argand plot for this find complex transfer functions and hence determine V

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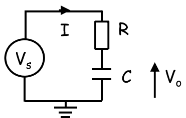
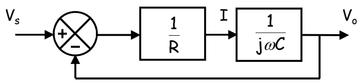
Transfer Function for RC Circuit



Z for resistor = R; for capacitor = $\frac{1}{j\omega C}$

Pot Divider $\frac{V_o}{V_s} = \dfrac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1}$

Or using block diagrams



So $\frac{V_o}{V_s} = \frac{\text{Forward}}{1 - \text{Loop}} = \dfrac{\frac{1}{R} * \frac{1}{j\omega C}}{1 - -\frac{1}{R} * \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1}$

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And So

$$\frac{V_o}{V_s} = \frac{1}{1 + j\omega CR} \quad \left|\frac{1}{1 + j\omega CR}\right| = \frac{1}{\sqrt{1 + \omega^2 C^2 R^2}} \quad \angle\frac{1}{1 + j\omega CR} = 0 - \tan^{-1}\omega CR$$

Suppose $R = 1k\Omega$, $C = 400\ \mu F$ and $V_s = 5 \sin(7t)$

$$\frac{V_o}{V_s} = \frac{1}{1 + j7 * 400 * 10^{-6} * 1 * 10^3} = \frac{1}{1 + j28 * 10^{2-6+3}} = \frac{1}{1 + j2.8}$$

$$\left|\frac{1}{1 + j2.8}\right| = \frac{1}{\sqrt{1 + 7.84}} = 0.336 : \angle\frac{1}{1 + j2.8} = -\tan^{-1}2.8 = -1.23\text{rad}$$

Hence $V_o = 5 * 0.336 \sin(7t - 1.23) = 1.68 \sin(7t - 1.23)$

http://www.reading.ac.uk/~shsmchlr/javascript/SinAndRC.html

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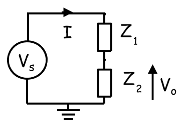
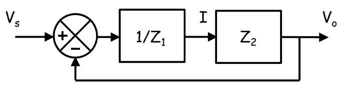
In terms of General Impedances





So $\frac{V_o}{V_s} = \frac{\text{Forward}}{1 - \text{Loop}} = \dfrac{\frac{1}{Z_1} Z_2}{1 - -\frac{1}{Z_1} Z_2} = \frac{Z_2}{Z_2 + Z_1}$

e.g If $V_s = 10\sin(3t)$, $Z_1 = j\omega 2 = j6$ and $Z_2 = 8$

$$\frac{V_o}{V_s} = \frac{8}{8 + j6} = \left|\frac{8}{\sqrt{64 + 36}}\right| \angle -\tan^{-1}\frac{6}{8} = 0.8\angle -0.644$$

Hence $V_o = 10*0.8\sin(3t - 0.644) = 8 \sin(3t - 0.644)$

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Summary

In this lecture : systems where its signals are sinusoids

- All same frequency
- May have different amplitude – may be phase shifted

Amplitude and Phase shift found using complex numbers

- Key point – use complex numbers to model calculus
  - We process by finding their modulus and argument
  - Shown working on electronics

Next week we develop this further,

- looking at other systems, with the same form of model

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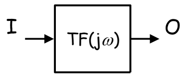
13 : Modelling Other Systems

Last week we saw how to model circuits

Including, as signals were sinusoids, how to represent integrators

Hence using complex numbers



If I = sin(ωt)

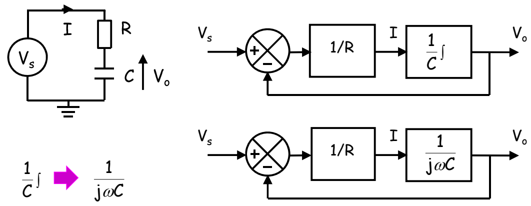
$O = |TF| \sin(\omega t + \angle TF)$

This week we develop this further and show

that the concept applies to other (non electronic) systems

First a reminder of the RC circuit from last week

RC Circuit Example

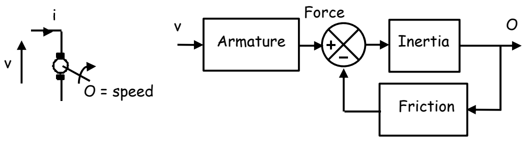


$\frac{1}{C}\int \Rightarrow \frac{1}{j\omega C}$

So $\frac{V_o}{V_s} = \frac{\text{Forward}}{1 - \text{Loop}} = \frac{\frac{1}{R} * \frac{1}{j\omega C}}{1 - -\frac{1}{R} * \frac{1}{j\omega C}} = \frac{1}{j\omega CR + 1}$

Motor

These concepts are not just applicable to electronics



v applied to armature circuit

→ current i

→ force to make motor move

→ motor moves, output velocity O

→ friction (which depends on velocity) opposes motion

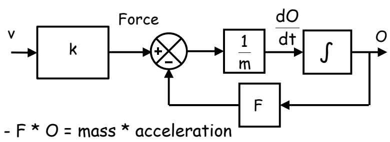
Incorporating the Inertia

Net Force = that due to Current (= v*k) – that due to Friction (F * O)

Net Force = mass * acceleration (Newton's 2nd Law)

Acceleration is change in (differential of), velocity

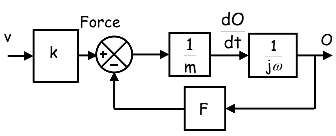
We want velocity, so we integrate acceleration for O



As v*k - F * O = mass * acceleration

$\text{acceleration} = \frac{dO}{dt} = \frac{\text{net force}}{\text{mass}} = \frac{v*k - F*O}{m}$

Forward over 1 minus Loop



Replace ∫ by 1/jω

$\frac{O}{v} = \frac{k * \frac{1}{m} * \frac{1}{j\omega}}{1 - -F * \frac{1}{m} * \frac{1}{j\omega}} = \frac{k}{jm\omega + F}$

So, if v is 4 sin(9t), k = 1 N/v, m = 0.2 kg, F = 0.1N/m (note units)

$\frac{O}{v} = \frac{1}{j0.2*9 + 0.1} = \frac{1}{j1.8 + 0.1}$

$\left|\frac{1}{j1.8 + 0.1}\right| = \frac{1}{\sqrt{1.8^2 + 0.1^2}} = 0.55 \qquad \angle\frac{1}{j1.8 + 0.1} = -\tan^{-1}\frac{1.8}{0.1} = -1.52$

So, O = 2.2 sin(9t - 1.52)

Rotational Movement

In the above example, the motor moves in straight line

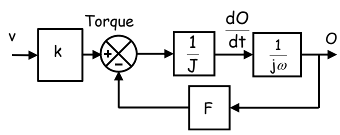
Its position (units m) changes, it moves with a given velocity (m/s)

It has mass (kg), it accelerates (ms^{-2}), due to force (N)

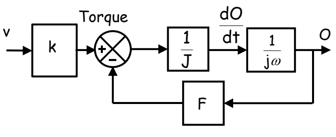
If the motor rotates, its angular position (units rad) changes

Have angular velocity (rad/s), angular acceleration (rad s^{-2}),

Due to torque (Nm), mass has moment of inertia J (kgm^2)



So



$$\frac{O}{v} = \frac{k * \frac{1}{J} * \frac{1}{j\omega}}{1 - -F * \frac{1}{J} * \frac{1}{j\omega}} = \frac{k}{jJ\omega + F}$$

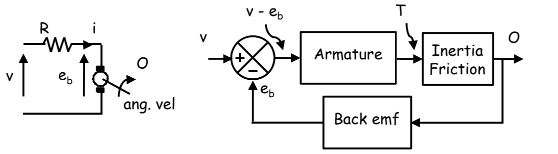
So, if I is 4 sin(9t), k = 1 Nm/v, J = 0.2 kg m², F = 0.1 Nm per rad/s

$$\frac{O}{v} = \frac{1}{j0.2*9 + 0.1} = \frac{1}{j1.8 + 0.1}$$

$$\left|\frac{1}{j1.8 + 0.1}\right| = \frac{1}{\sqrt{1.8^2 + 0.1^2}} = 0.55 \qquad \angle \frac{1}{j1.8 + 0.1} = -\tan^{-1}\frac{1.8}{0.1} = -1.52$$

So, O = 2.2 sin(9t - 1.52)

Permanent Magnet Motor



v applied to armature circuit: motor turns e_b 'back emf' generated

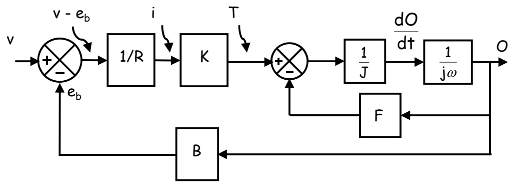
Difference between v and e_b → V across R → current, i

i → torque T (= K*i) → angular velocity O (note inertia/friction)

O → e_b (= B*O)

Slightly more complicated model ..

Full Block Diagram



Use Forward over 1 - Loops

$$\frac{O}{v} = \frac{\frac{1}{R} * K * \frac{1}{J} * \frac{1}{j\omega}}{1 - -B * \frac{1}{R} * K * \frac{1}{J} * \frac{1}{j\omega} - -F * \frac{1}{J} * \frac{1}{j\omega}}$$

Simplify

$$\frac{O}{v} = \frac{\frac{1}{R} * K * \frac{1}{J} * \frac{1}{j\omega}}{1 - -B * \frac{1}{R} * K * \frac{1}{J} * \frac{1}{j\omega} - -F * \frac{1}{J} * \frac{1}{j\omega}}$$

$$\frac{O}{v} = \frac{\frac{K}{j\omega RJ}}{1 + \frac{BK}{j\omega RJ} + \frac{F}{j\omega J}} \qquad \text{Multiply by } \frac{j\omega RJ}{j\omega RJ}$$

$$\frac{O}{v} = \frac{K}{j\omega RJ + BK + FR}$$

This is in same form as in previous examples $\frac{O}{v} = \frac{K}{a + j\omega b}$

So if have values for K, R, J, B and know v, can work out O

Lecture 13 In Class Exercise

For motor $\frac{O}{v} = \frac{K}{j\omega RJ + BK + FR}$

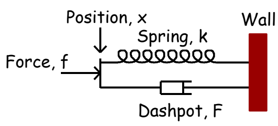
Suppose K = 0.1, R = 1k, J = 0.05, F = 0.01, B = 0.1 and v = 10sin(0.1t)

What is O?

Mechanical Example

Force f applied to spring attached to wall

'Dashpot' represents friction opposing motion



When spring compressed by x, force k*x opposes (Hooke's Law)

Frictional force = F * differential of x also opposes

As $\int \equiv \frac{1}{j\omega}$, So $\frac{d}{dt} \equiv j\omega$. Hence dashpot force is F*jω * x

Hence f = k*x + F*jω * x = (k + jωF) x

$$\text{Hence } \frac{x}{f} = \frac{1}{k + j\omega F}$$

Standard Model

For RC circuit: $\frac{V_o}{V_s} = \frac{1}{1 + j\omega CR}$

First motor: $\frac{O}{I} = \frac{1}{jm\omega + F}$

Perm. magnet motor: $\frac{O}{v} = \frac{K}{j\omega RJ + BK + FR}$

Spring Sys: $\frac{x}{f} = \frac{1}{k + j\omega F}$

For convenience, it is useful to have a standard model $\frac{K}{1 + j\omega T}$

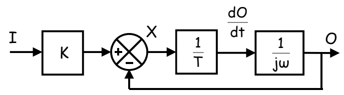
For RC circuit, already there : K = 1 and T = RC

For first motor: $\frac{O}{I} = \frac{1}{jm\omega + F} = \frac{1/F}{1 + jm\omega/F}$

For perm. magnet motor: $\frac{O}{v} = \frac{\frac{K}{BK + FR}}{1 + j\omega\frac{RJ}{BK + FR}}$

For Spring Sys: $\frac{x}{f} = \frac{\frac{1}{k}}{1 + j\omega\frac{F}{k}}$

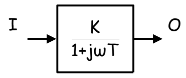
Block Diagram For This Form



Using Forward over 1-loop

$$\frac{O}{I} = \frac{K * \frac{1}{T} * \frac{1}{j\omega}}{1 - -\frac{1}{T} * \frac{1}{j\omega}} = \frac{\frac{K}{j\omega T}}{1 + \frac{1}{j\omega T}} = \frac{K}{1 + j\omega T}$$

So System reduces to



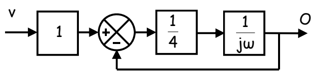
http://www.reading.ac.uk/~shsmchlr/javascript/SysAndSin.html

Permanent Magnet Motor in this form

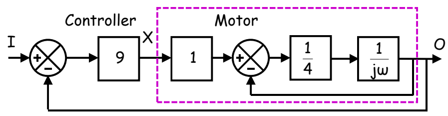
$$\frac{O}{v} = \frac{K}{j\omega RJ + BK + FR}$$

Let K = 0.1, B = 0.9, R = 100, F = 0.0001, J = 0.004

$$\frac{O}{v} = \frac{0.1}{j\omega 100*0.004 + 0.09 + 0.01} = \frac{0.1}{j\omega 0.4 + 0.1} = \frac{1}{1 + j\omega 4}$$



Connect Motor in Control System



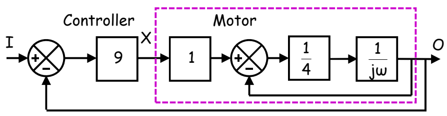
Use forward over 1 minus each loop

$$\frac{O}{I} = \frac{9*1*\frac{1}{4}*\frac{1}{j\omega}}{1 - -\frac{1}{4}*\frac{1}{j\omega} - -9*1*\frac{1}{4}*\frac{1}{j\omega}} = \frac{\frac{9}{j\omega 4}}{1 + \frac{1}{j\omega 4} + \frac{9}{j\omega 4}} = \frac{9}{10 + j\omega 4} = \frac{0.9}{1 + j\omega 0.4}$$

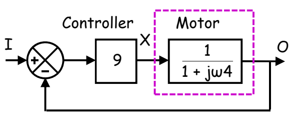
Still in standard form.

Key point : K and T of original motor changed

Or Can Simplify First



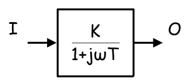
But motor is $\frac{1}{1 + j\omega 4}$



$$\frac{O}{I} = \frac{9*\frac{1}{1 + j\omega 4}}{1 + 9*\frac{1}{1 + j\omega 4}} = \frac{9}{1 + j\omega 4 + 9} = \frac{9}{10 + j\omega 4} = \frac{0.9}{1 + j\omega 0.4}$$

Same answer. Same form

Key Point to Note



$$\left|\frac{O}{I}\right| = \frac{K}{\sqrt{1 + \omega^2 T^2}} \qquad \angle\frac{O}{I} = \tan^{-1}0 - \tan^{-1}\omega T = -\tan^{-1}\omega T$$

if I = sin(ωt)

$$O = \frac{K}{\sqrt{1+\omega^2 T^2}}\sin(\omega t + \phi) \quad \text{where } \phi = -\tan^{-1}(\omega T)$$

Modulus of transfer function — argument of transfer function

If interested, two slides after lecture summary confirm this :
Also shows that complex numbers makes it easier

Summary

In this lecture we have further investigated system models

We assume the input is a sinusoid and model integration by $1/j\omega$

For the RC circuits and the motors the same form of model found

When we put feedback round it, the same form of model appears

$$\frac{K}{1+j\omega T}$$

Can find output using modulus and argument of this

Later we develop this by considering what happens when ω changes

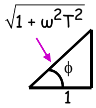
Next week : how to model systems when inputs are not sinusoids…

Confirmation – if interested

$I = \sin(\omega t)$; $O = \frac{K}{\sqrt{1+\omega^2T^2}}\sin(\omega t - \phi)$ where $\phi = \tan^{-1}(\omega T)$

NB $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

So $\sin(\omega t-\phi) = \sin(\omega t)\cos(\phi) - \cos(\omega t)\sin(\phi)$

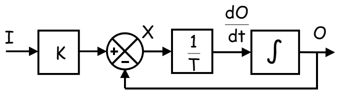


$$\cos(\phi) = \frac{1}{\sqrt{1+\omega^2T^2}} \quad \sin(\phi) = -\frac{\omega T}{\sqrt{1+\omega^2T^2}}$$

$$O = \frac{K}{\sqrt{1+\omega^2T^2}}\left(\sin(\omega t)\frac{1}{\sqrt{1+\omega^2T^2}} - \cos(\omega t)\frac{\omega T}{\sqrt{1+\omega^2T^2}}\right)$$

$$O = \frac{K}{1+\omega^2T^2}(\sin(\omega t) - \omega T\cos(\omega t))$$

Concluded



$$= \frac{K(\sin(\omega t) - \omega T\cos(\omega t))}{1+\omega^2T^2}$$

$$\frac{dO}{dt} = \frac{K*I - O}{T} = \frac{K*\sin(\omega t) - \frac{K}{1+\omega^2T^2}(\sin(\omega t) - \omega T\cos(\omega t))}{T}$$

$$= \frac{K\omega^2T^2\sin(\omega t) + K\omega T\cos(\omega t)}{T(1+\omega^2T^2)}$$ integrate this to get O

$$O = \frac{K\omega^2T^2\frac{-1}{\omega}\cos(\omega t) + K\omega T\frac{1}{\omega}\sin(\omega t)}{T(1+\omega^2T^2)} = \frac{-K\omega T\cos(\omega t) + K\sin(\omega t)}{(1+\omega^2T^2)}$$

Yes - complex numbers easier

Lecture 14 - Frequency Response

We have also analysed systems where the input is a sinusoid

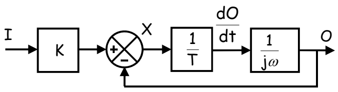
- For this we use $1/j\omega$ for ∫
- As a result the transfer function is a complex number
  - This has modulus and argument
  - Which can represent gain and phase shift of system
- Hence, if we know the input sinusoid we can find its output

Also useful to see how gain and phase vary with frequency

Today we plot this variation –

- Using one graph, then two related graphs
  - This is very useful for analysing systems

First Order Systems - Reminder



$$\frac{O}{I} = \frac{K\frac{1}{T}\frac{1}{j\omega}}{1 - -\frac{1}{T}\frac{1}{j\omega}}$$

$$\frac{O}{I} = \frac{K}{1+j\omega T}$$

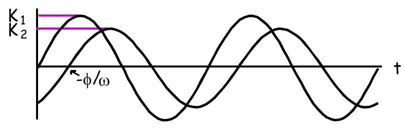
$$\left|\frac{O}{I}\right| = \frac{K}{\sqrt{1+\omega^2T^2}} \qquad \angle\frac{O}{I} = \tan^{-1}0 - \tan^{-1}\omega T = -\tan^{-1}\omega T$$

if $I = K_1 * \sin(\omega t)$

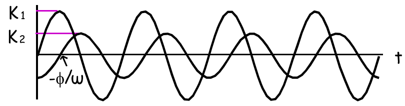
$$O = K_1 * \frac{K}{\sqrt{1+\omega^2T^2}}\sin(\omega t + \phi) \quad \text{where } \phi = -\tan^{-1}(\omega T)$$

Modulus of transfer function — argument of transfer function

Input, $K_1 \sin(\omega t)$ Output $K_2 \sin(\omega t+\phi)$



Higher freq : smaller K_2, larger phase shift -ϕ



More useful, plot graphs showing how gain and phase vary with ω

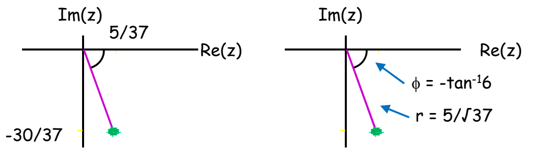
In fact we see how the transfer function varies with ω

Transfer Function at different ω

For System $\frac{K}{1+j\omega T}$ if K = 5, T = 2, ω = 3, TF is $\frac{5}{1+j6}$

TF = $\frac{5}{\sqrt{37}}\angle -\tan^{-1}6$ or $\frac{5}{37} - j\frac{30}{37}$

Can Plot on Argand Diagram … in Cartesian or Polar form



But if ω = 1, System $\frac{5}{1+j2} = \frac{5}{\sqrt{5}}\angle -\tan^{-1}2$ or 1-j2

MatLab Calculations

```
>> K = 5; T = 0.2; w = 1;
>> tf = K/(1+j*w*T)      % find TF at w = 1
tf = 4.8077 - 0.9615i
```

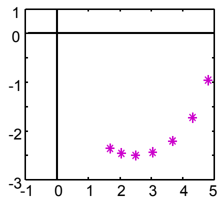
Then use real(tf), imag(tf) or abs(tf), angle(tf)

We need to make such calculations over large range … Say 0.1 .. 1000

```
>> tf = K./(1+j*[0.1, 1, 10, 100, 1000]*T)   %note ./ to get TF as vector
tf = 4.9980 - 0.1000i  4.8077 - 0.9615i  1.0000 - 2.0000i
     0.0125 - 0.2494i  0.0001 - 0.0250i
>> abs(tf)    gives  4.9990   4.9029   2.2361   0.2497   0.0250
>> angle (tf) gives  -0.0200  -0.1974  -1.1071  -1.5208  -1.5658
```


MatLab can plot for different ω

```
>> K = 5; T = 0.2;
>> w = [1,2,3,4,5,6,7];   % some w
>> tf = K./(1+j*w*T)      % calc TF at each.
tf =
  4.8077 - 0.9615i   4.3103 - 1.7241i
  3.6765 - 2.2059i   3.0488 - 2.4390i
  2.5000 - 2.5000i   2.0492 - 2.4590i
  1.6892 - 2.3649i
>> plot(real(tf), imag(tf), '*',  [-1 5], [0 0], [0 0],[-3 1]);
                % plot *'s and axes
```



Note *'s get closer : linearly spaced ω not best :

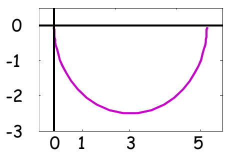
So need to plot over larger ω range .. And not linearly spaced

Continuous Plot in MATLAB

Better if ω logarithmically not linearly spaced, and join dots

Use logspace(a, b) : generates 50 values between 10^a and 10^b.

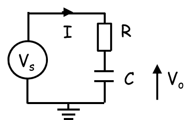
```
>> w = logspace(-1,2);         % ω from 0.1 to 100 : ok this sys
>> tf = K./(1+j*w*T);          % trans func at all ω
>> plot(real(tf),imag(tf), ... % plot 'locus'
   [-0.5,K+0.5], [0 0],[0, 0], [-K/2-0.5, 0.5]);
```

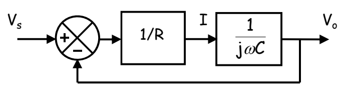


Shows how gain and phase vary with frequency on one plot – System's Frequency Response

NB this is a semi-circle radius K/2, origin K/2,0

14 – In Class Exercise



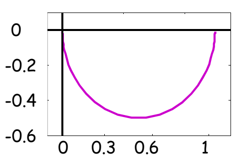


$\frac{V_o}{V_s} = \frac{1}{1+j\omega CR}$ Sketch its polar plot.

At very low ω, gain is 1

So is semi-circle

Radius 0.5, org 0.5, 0



Two Plots

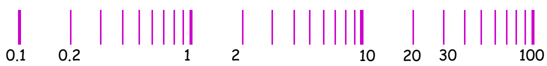
The single plot shows how gain and phase vary together

Can be of interest to see how gain and phase each vary with ω

But saw polar plot calculated from ω = 0 to 100 or even 1000 rad/s

Information ω = 0.1 : 1 just as important as from 1 :10, 10 : 100

So the ω axis is plotted using logarithmic scales:



As gain varies a lot from 10..0.05 also use logarithmic gain scale

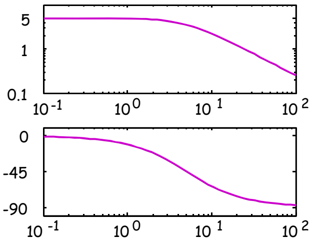
Phase varies from 0 to -90^o … so use linear scale

So plot log(gain) vs log(ω) and phase vs log(ω)

Bode Plot for 5/(1+jw 0.2)

Given have already calculated w and tf ...

```
>> subplot(2,1,1); loglog(w,abs(tf));
>> subplot(2,1,2); semilogx(w, angle(tf)*180/pi);
```



Plots log Gain v log ω

Plots Phase v log ω

Approximate Behaviour

Recall: $TF = \frac{K}{1 + j\omega T}$: Gain = $\frac{K}{\sqrt{1+\omega^2T^2}}$; Phase = $-\tan^{-1}(\omega T)$

Suppose ωT very small (« 1), so treat $1 + j\omega T$ as 1

$TF = \frac{K}{1}$, so Gain = K; Phase = 0

Suppose ωT very big (» 1), so approximate $1 + j\omega T$ as $j\omega T$

$TF = \frac{K}{j\omega T}$, so Gain = $\frac{K}{\omega T}$; Phase = -90^O

These define behaviour at very low and very high freq

Actual behaviour in between these, eg

At $\omega T = 1$ $TF = \frac{K}{1+j}$: Gain = $\frac{K}{\sqrt{1+1}} = \frac{K}{\sqrt{2}}$; Phase = -45^O

Plotting Approximate Behaviour

At Low Freqs, approximate Gain = K and Phase = 0

Plot log(Gain) vs log(ω) log (K) is constant

So Plot horizontal straight line : its slope is 0

At High Freqs, Gain = $\frac{K}{\omega T}$ and Phase = -90^O

$$\log\left(\frac{K}{\omega T}\right) = \log\left(\frac{K}{T}*\omega^{-1}\right) = \log\left(\frac{K}{T}\right) + \log\left(\omega^{-1}\right) = \log\left(\frac{K}{T}\right) - \log(\omega)$$

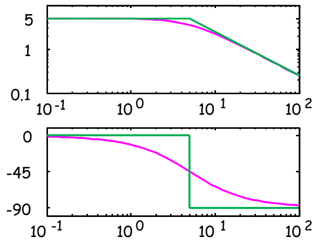
So Plot is const – log(ω) plotted vs log(ω)

Like plotting c - x vs x ie straight line slope -1

We call these high and low freq lines asymptotes

Asymptotic Plot (actual superimposed)

Asymptotic TF : K and $\frac{K}{j\omega T}$ ie 5 and $\frac{5}{j\omega 0.2} = \frac{25}{j\omega}$



G = K and K/ωT

line slopes 0 and -1

Meet at ωT = 1 : ω = 5

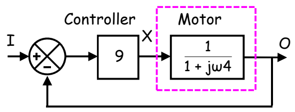
P = 0 and P = -90^O

Change at ω = 5

Actual plots start on low freq asymptotes and end on high f lines

Given TF, easy to sketch asymptotes and then add actual

Freq Response of Motor in Feedback

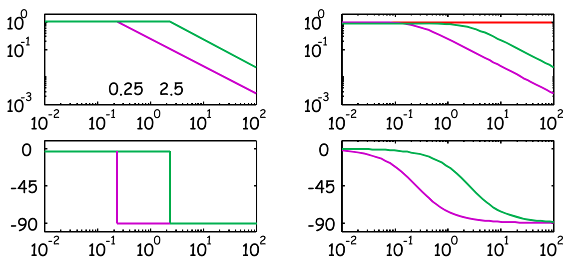


$$\frac{O}{I} = \frac{9*\frac{1}{1+jw4}}{1+9*\frac{1}{1+jw4}} = \frac{9}{1+jw4+9} = \frac{9}{10+jw4} = \frac{0.9}{1+jw0.4}$$

With feedback, corner freq moved from 0.25 to 2.5 rad/s

Let's plot superimposed $\frac{1}{1+jw4}$ and $\frac{0.9}{1+jw0.4}$

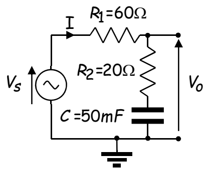
Graphs ... Asymptotes and actual



With feedback, Gain higher over more frequencies

Phase near 0 over more frequencies

Phase Lag System

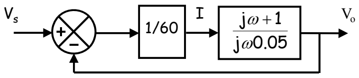


Treat like Potential Divider …

$(V_s - V_o) /\ R_1$ determines current

Current through R_2 + C sets V_o

$$R_2 + \frac{1}{j\omega C} = \frac{j\omega CR_2 + 1}{j\omega C} = \frac{j\omega + 1}{j\omega 0.05}$$



$$\frac{V_o}{V_s} = \frac{\frac{1}{60} * \frac{j\omega+1}{j\omega 0.05}}{1 + \frac{1}{60} * \frac{j\omega+1}{j\omega 0.05}} = \frac{j\omega+1}{j\omega 3 + j\omega + 1} = \frac{j\omega+1}{j\omega 4 + 1}$$

p73 RJM 08/12/15 SE1CY15 - Feedback - Part D

Continued

$$\frac{V_o}{V_s} = \frac{j\omega + 1}{j\omega 4 + 1}$$ Two corner freqs : $\frac{1}{4}$ and 1 rad/s

We can plot this by generating asymptotes as before

For each 1+jωT, approx as 1 before ω = 1/T, and jωT after

So consider what happens before $\frac{1}{4}$, from $\frac{1}{4}$ to 1, then after 1

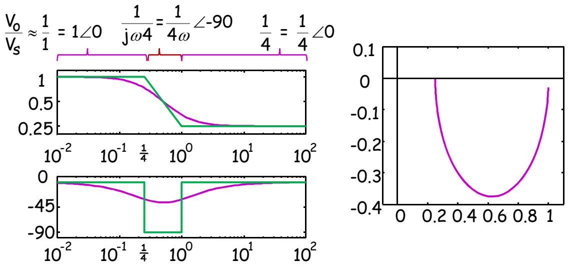
So ω < 0.25: $\frac{V_o}{V_s} \approx \frac{1}{1} = 1$; Gain = 1; Phase = 0

0.25 < ω < 1: $\frac{V_o}{V_s} \approx \frac{1}{j\omega 4}$; Gain = $\frac{1}{\omega 4}$; Phase = -90

ω > 1: $\frac{V_o}{V_s} \approx \frac{j\omega}{j\omega 4} = \frac{1}{4}$; Gain = 0.25; Phase = 0

p74 RJM 08/12/15 SE1CY15 - Feedback - Part D

Bode and Polar Plots



In mid freqs, around √(1*4) = 0.5 rad/s, phase is -ve, so V_o 'lags' V_s

Such a 'phase lag' circuit can be used for control

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Summary

In this lecture we consider plotting frequency response

That is, how does system transfer function vary with ω.

We first plotted these on one graph

- Calculating gain/phase or real + j imag points and joining them

Then we plotted log(gain) vs log(ω) and phase vs log(ω)

- Both the actual graphs and the asymptotes
- Asymptotes useful for sketching such graphs
- Saw that with phase lead circuit.

Next week we move to considering how system changes with time

- We will do frequency response of second order systems later

p76 RJM 08/12/15 SE1CY15 - Feedback - Part C

15 – Time Response of Systems

We modelled different dynamic systems which include integrators

We have determined their output

- If the input is a step, the output is a constant value
  - determined by when the input to the integrator is 0
- If the input is a sinusoid, the output is also a sinusoid
  - determined by modelling the system using complex numbers

This answered one of two questions posed

- The other is how does the output get to its final value.

Strictly so far we have determined the steady state response

- We now work towards how it gets there: the transient response

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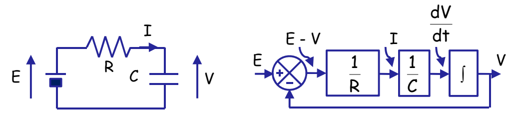
Consider the RC Circuit



From block diagram, $\frac{dV}{dt} = I * \frac{1}{C} = (E - V) * \frac{1}{R} * \frac{1}{C}$

When E is connected at t = 0, in effect it is a step input

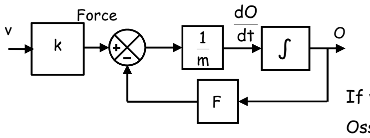
V will reach the final value of E (which is constant after t=0)

Final value means V not changing, ie dV/dt = 0

Clearly this is when I = 0, which is when E – V = 0, or V = E

p78 RJM 08/12/15 SE1CY15 - Feedback - Part C

Now consider a motor and general system

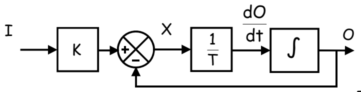


From block diagram, $\frac{dO}{dt} = \frac{1}{m}*(v*k - F*O)$

If v is step applied at t = 0

Oss when v * k - F * O = 0,

or O = v * k /F

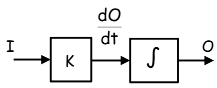


$\frac{dO}{dt} = \frac{1}{T}*X = \frac{I*K - O}{T}$

I*K - Oss = 0 or Oss = I*K

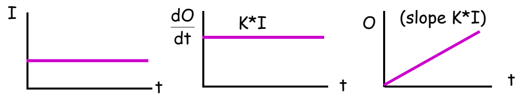
We argue intuitively how O or V gets to steady state .. Use integrators

If Put Constant Into An Integrator:

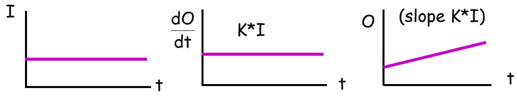


If O is ∫ block output

$\frac{dO}{dt}$ must be block input



Or, if K smaller, and O is not 0 initially



At time t, O = (value of O at t = 0) + K * I * t

What if I series of constants?

Suppose K = 2 and at t = 0, O = 0,

I = 1 from 0..5s,

I = -0.5 from 5..8s,

and I thereafter = 0.25

After 5s,

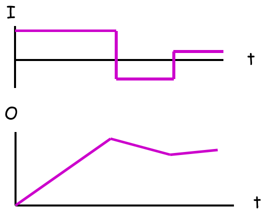
O = 0 + 1*2*5 = 10

After 8s,

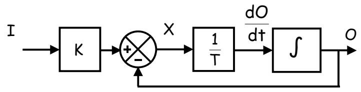
O = 10 + -0.5*2*3 = 7

After 10s,

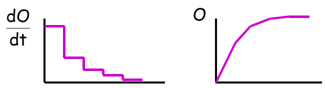
O = 7 + 0.25*2*2 = 8



If Put Feedback Round Integrator



O = 0
at t=0;
I is 1



Initially O is 0, so X & dO/dt are big, so O rises much

X and dO/dt now smaller, so O rises but by less

X and dO/dt even smaller, so O rises by even less

Eventually O = I, so X = dO/dt = 0, and hence O is constant

Step Responses

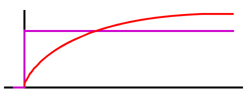
Often we analyse systems as follows:

At time t = 0, output O = 0, input turned on

I goes 0 to value : is a 'step' (unit step if value = 1)

O moves smoothly from 0 to a final value

Called an exponential lag (it lags behind input)

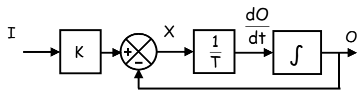


Output O
Input I
time, t

O reaches a (constant) value – its *steady state;*

How gets there is the (exponential) *transient* response

Formalising The Response



X = I * K - O

So $\frac{dO}{dt} = \frac{1}{T}(K*I - O)$

At steady state, dO/dt = 0, that is K*I – O = 0

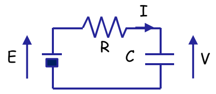
So steady state value of O is K * I = K if I is unit step

In Maths, you will learn to show transient response is $-Ke^{-t/T}$

Done by solving $\frac{dO}{dt} = -\frac{O}{T}$ (i.e. ignoring input I)

The complete response is $O = K - Ke^{-t/T}$

So for RC circuit



$$\frac{dV}{dt} = \frac{E - V}{R*C}$$

Steady state found by solving $\frac{dV}{dt} = 0$ So $V_{ss} = E$

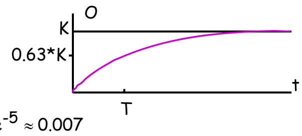
Transient found by solving $\frac{dV}{dt} = -\frac{V}{R*C}$ So $V_t = -E\ e^{-t/RC}$

So complete response is $V = E - E\ e^{-t/RC}$

Fundamental Point

If system model $\frac{dO}{dt} = \frac{1}{T}(K*I - O)$; I = unit step; O at t= 0 is 0

Then $O = K - Ke^{-t/T}$



$e^{-1} \approx 0.37$; $e^{-3} \approx 0.05$; $e^{-5} \approx 0.007$

At t = T, $K - K\,e^{-1} = K * 0.63$ (O is 63% of final value)

At t = 3T, $K - K\,e^{-3} = K * 0.950$ (O is 5 % from final value)

At t = 5T, $K - K\,e^{-5} = K * 0.993$ (O <1% from final value)

http://www.reading.ac.uk/~shsmchlr/javascript/SysAndStep.html

If you are interested …

We have stated that I is a unit step, = constant for t>0

$$O(t) = K - K * e^{-t/T}$$

Also $\frac{dO}{dt} = \frac{1}{T}(K*I - O)$

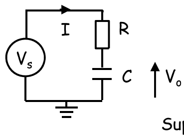
This demonstrates that these are consistent

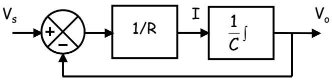
$$\frac{dO}{dt} = \frac{dK}{dt} - \frac{dK*e^{-t/T}}{dt} = 0 - -\frac{1}{T}K*e^{-t/T} = \frac{K}{T}e^{-t/T}$$

$$\frac{1}{T}(K*I - O) = \frac{1}{T}\left(K - K + K*e^{-t/T}\right) = \frac{K}{T}e^{-t/T}$$

Hence expression for O is a solution to the diff eqn

Electronic Example





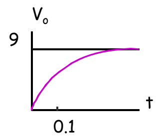
Suppose V_s is Step = 9V; R = 5kΩ and C = 20μF

$$\frac{dV_o}{dt} = \frac{1}{5*10^3*20*10^{-6}}*(9 - V_o)$$

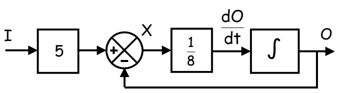
$$\frac{dV_o}{dt} = \frac{1}{0.1}*(9 - V_o)$$

In standard form, so if V_o = 0 at t=0

$V_o = 9 - 9\,e^{-t/0.1}$



Lecture 15 - In Class Exercise



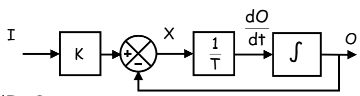
Derive an expression for dO/dt in standard form.

Suppose I = step height 1 applied at t = 0 when O = 0:

Sketch a graph of O versus t : label final value and time constant.

Effect of Feedback – to Step Input

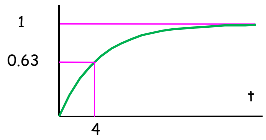
System could be motor on its own: open loop



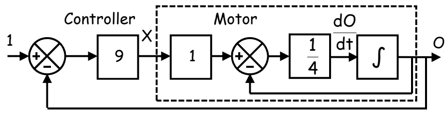
Modelled by $\frac{dO}{dt} = \frac{K*I - O}{T}$

Suppose I = 1, K = 1 and T = 4s $\frac{dO}{dt} = \frac{1*1 - O}{4} = \frac{1 - O}{4}$

In standard form, so can say:
Steady State value 1
Reaches 63% of 1 at 4s
Within 1% of 1 by 20s



Now This Motor In Control System



Controller output = 9*(1-O), X = 9*(1-O) – O, so

$$\frac{dO}{dt} = \frac{1}{4}*(9*(1-O)-O) = \frac{9-9*O-O}{4} = \frac{9-10*O}{4}$$

Not in right form as O multiplied by 10: so divide by 10

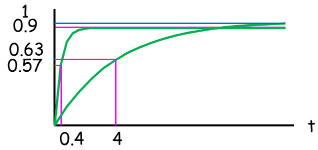
$$\frac{dO}{dt} = \frac{\frac{9}{10} - O}{\frac{4}{10}} = \frac{0.9-O}{0.4}$$

Final value 0.9. Reach 63% of this at t = 0.4s

Open and Closed Loop Responses

Open loop: $\frac{dO}{dt} = \frac{1-O}{4}$ Final value 1, Time Const 4

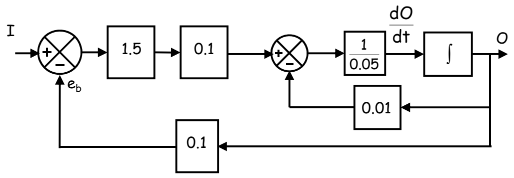
Closed loop: $\frac{dO}{dt} = \frac{0.9-O}{0.4}$ Final value 0.9, Time Const 0.4



See Frequency Response Lectures: same example : same 0.9 & 0.4

http://www.reading.ac.uk/~shsmchlr/javascript/SysAndStep.html

Another Example - Motor



$$\frac{dO}{dt} = \frac{1}{0.05}((I-0.1*O)*0.15 - 0.01*O)$$

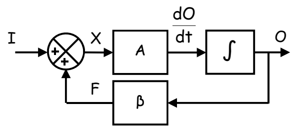
$$\frac{dO}{dt} = \frac{1}{0.05}(I*0.15 - 0.025*O) \quad * \frac{1/0.025}{1/0.025}$$

$$\frac{dO}{dt} = \frac{1}{2}(I*0.6 - O)$$

Now in standard form, K = 0.6, T = 2 so if I step

$O(t) = 0.6 - 0.6 * e^{-t/2}$

Aβ Feedback System With Integrator



$$\frac{dO}{dt} = A(I + \beta O)$$

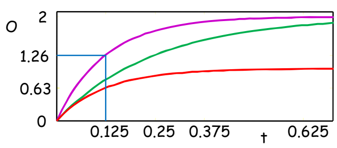
For standard form, rejig so $\frac{dO}{dt} = \frac{1}{T}(K*I - O)$

$$\frac{dO}{dt} = -A(-I - \beta O) = -A\beta(-\frac{I}{\beta} - O)$$

So K = -1/ β and T = -1/Aβ;

Feedback gain β sets final value, loop gain Aβ the speed

Some Example Responses



I = 1
a) A = 16, β = -0.5
b) A = 8, β = -0.5
c) A = 8, β = -1
Want final value & t when 63% of it

	A	β	Final Value	t at 63%	I/-β	1/-Aβ
a	16	-0.5	2	0.125	2	1/8
b	8	-0.5	2	0.25	2	1/4
c	8	-1	1	0.125	1	1/8

Summary

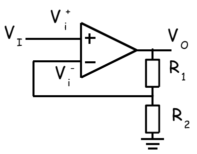
We have investigated the step response of a first order system
- Have seen steady state response – being a constant
- And transient response – being an exponential

The time constant indicates the speed of response :
- reaches 63% of final value at t = time constant
- within 1% of final value at 5 * this value

We have seen that putting feedback round such systems, speeds up the response

Next week, we introduce the s-operator
- to simplify diagrams/analysis .. Be consistent with jω

Lecture 11 - After Class Exercise



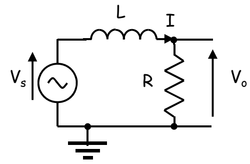
Find V_o if V_I = 1V, OpAmp gain A = 1000, R_1 = 80kΩ and R_2 = 20kΩ.

Note, find the exact value, and compare with the approximate one.

By how much does V_o change, if A increases by 10% to 1100?

Does the system have negative feedback?

Lecture 12 – After Class Exercise



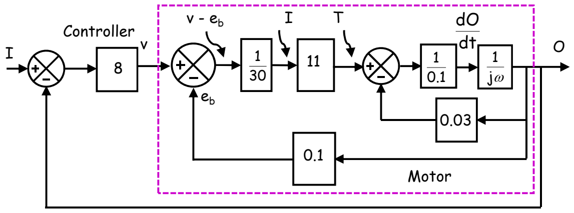
If V_L is the voltage across the inductor, I = $V_L/j\omega L$.

As such, draw a block diagram of the circuit.

Hence work out the circuit transfer function.

What is V_O if L = 0.5H, R = 20Ω and V_S is 7 sin(8t)?

Lecture 13 – After Class Exercise



The above shows a permanent magnet motor in a control system

Use forward / (1 – loops) to find the transfer function O/I.

What is O if I is 9 sin (40 t)?

