CONTROL USING MAXIMUM AVAILABLE FEEDBACK

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Overview

Maximum Available Feedback is max loop gain over a specified bandwidth for given stability margins, in a single loop feedback system

Developed by Bode for Electronic Amplifiers, using Asymptotic Approximations

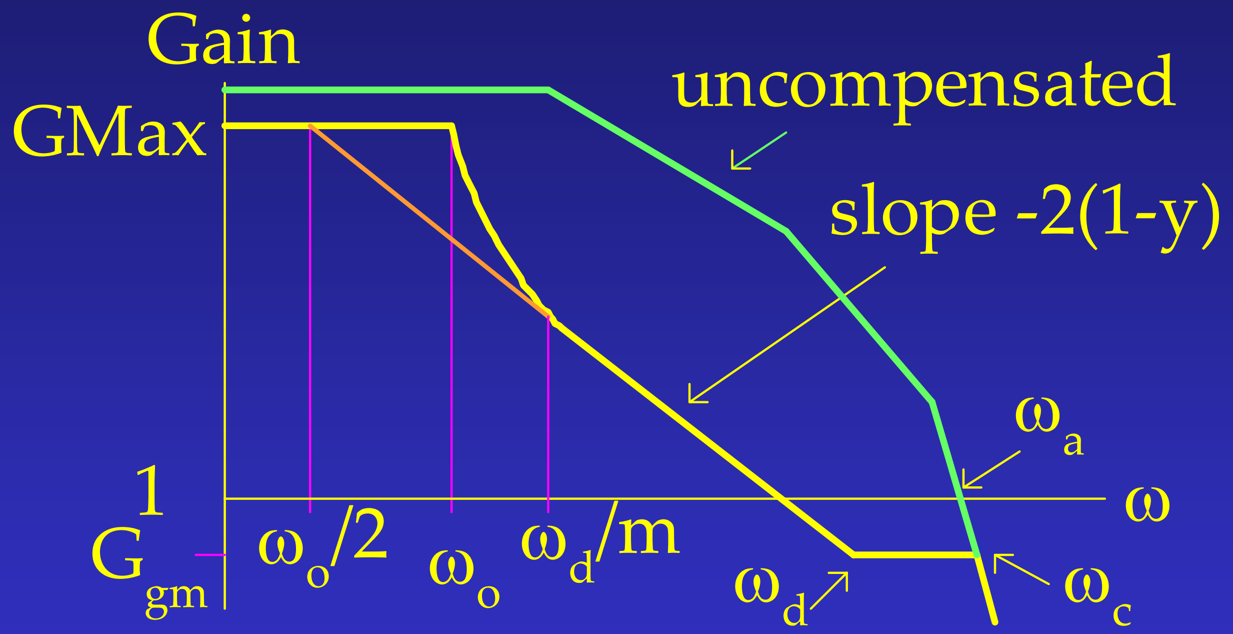
But, appropriate for Control to have high loop gain

Also, is a good example of a non-trivial controller

Paper shows how to approach design from a control perspective, using novel analysis of Bode's asymptotes

Frequency Shape for Bode's Design

Uncompensated System: gain = 1 @ ω_a ; slope is -n



Specify

ω_o= bw

x = Gain Margin

y = Rel Phase Margin (PM/180)

Slope -2(1-y) → Phase = -180 + PM;

'Bode Step' $\omega_d..\omega_c$: cancel phase due to –n slope

Gain 'curves up' to double bandwidth

Loop Transfer Function – 3 Parts

Design produces transfer function round loop
- Curved Part : low freq response
    - Bode’s irrational element awkward, so use
    - Second Order Element, corner freq ω_o
    - In effect slope -2 from ω_o to -2(1-y) slope
- Lead Lag(s) to approximate slope -2(1-y)
    - from ω_d / *m* to Bode Step (at ω_d)
- Double Lead for Bode Step at ω_d
- Then n Lags at ω_c

Controller in Series with UnComp for Loop TF

Transfer Functions

$$\text{Loop TF}: \frac{\text{GMax}}{s^2/\omega_o^2 + s/\omega_o + 1}\,\frac{1+s/\omega_1}{1+s/\omega_2}\,\frac{(1+s/\omega_d)^2}{(1+s/\omega_c)^n}$$

$$\text{GMax (in dB)} = 40(1-y)\log_{10}\left(\frac{4(1-y)}{n}10^{\frac{x}{20n}}\frac{\omega_a}{\omega_o}\right) - x$$

If, over bandwidth, slope to be -1, so $O_{ss} = 0$ to step

$$\text{Loop TF}: \frac{\text{GMax}*\omega_o}{s(1+s/\omega_o)}\,\frac{1+s/\omega_1}{1+s/\omega_2}\,\frac{(1+s/\omega_d)^2}{(1+s/\omega_c)^n}$$

If -2(1-y) slope over large range, use more LeadLags

Can also cope with sampling and hence Time Delay

Applying to Control

Electronics: large d.c. gain, so ω_a at high freq, after most corner freqs, and order, n, is high method needed to stabilise system

Control : d.c. gain may be less than 1: no ω_a or small, most corner freqs after ω_a

Also, often specify Control in terms of step response

BUT, good for Control to have high loop gain

(output largely unaffected by disturbance or by changes in parameters of device under control)

Approach

Loop TF will have high d.c. loop gain

To implement need an amplifier

Thus include in ‘uncompensated system’ both the device to be controlled AND a ‘virtual’ amplifier

Gain of the amplifier affects ω_a

Also approx relationship exists between TimeToPeak (T_{pk}) to Step and ω_d (and hence ω_a) in terms of phase margin which is related to %overshoot (%os)

So, from %os and T_{pk}, assuming typical gain margin, estimate ω_a and gain of virtual amplifier → design

Details

Uses second order correlations; ζ = damping ratio

PM ~ 100 ζ $\qquad T_{pk} = \frac{\pi}{\omega_{rf}\sqrt{1-\zeta^2}} \qquad \%os = 100 * e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$

ω_{rf} where closed loop gain max; when slope is -2(1-y)

Assume Loop TF is $\frac{K}{(j\omega)^{2(1-y)}} \qquad K = 10^{-\frac{x}{20}} * \omega_d^{2(1-y)}$

$$\omega_{rf} \approx \omega_d \left(10^{-\frac{x}{20}} \cos(\pi y)\right)^{\frac{1}{2(1-y)}} : \text{typically} \approx 0.2\,\omega_d$$

Choose suitable ω_o best if m, freq range of -2(1-y), > 50

Examples

Speed Control of Motor and associated Power Amp

$$H(s) = \frac{2}{(1+s/6)(1+s/40)(1+s/80)}$$

GM = 15dB, PM = 45^{O} (~ 20% o/s) $T_{pk} = 0.1$s.

ω_{rf}~35 rad/s ω_d~140 rad/s ω_c~280 rad/s ω_a~158 rad/s

Virtual amplifier: gain = 122 corner freq = 600 rad/s.

If $\omega_o = 0.3$ rad/s, -2(1-y) over freq range $58 = 2^{1-{}^{1}\!/_{y}}\dfrac{\omega_d}{\omega_o}$

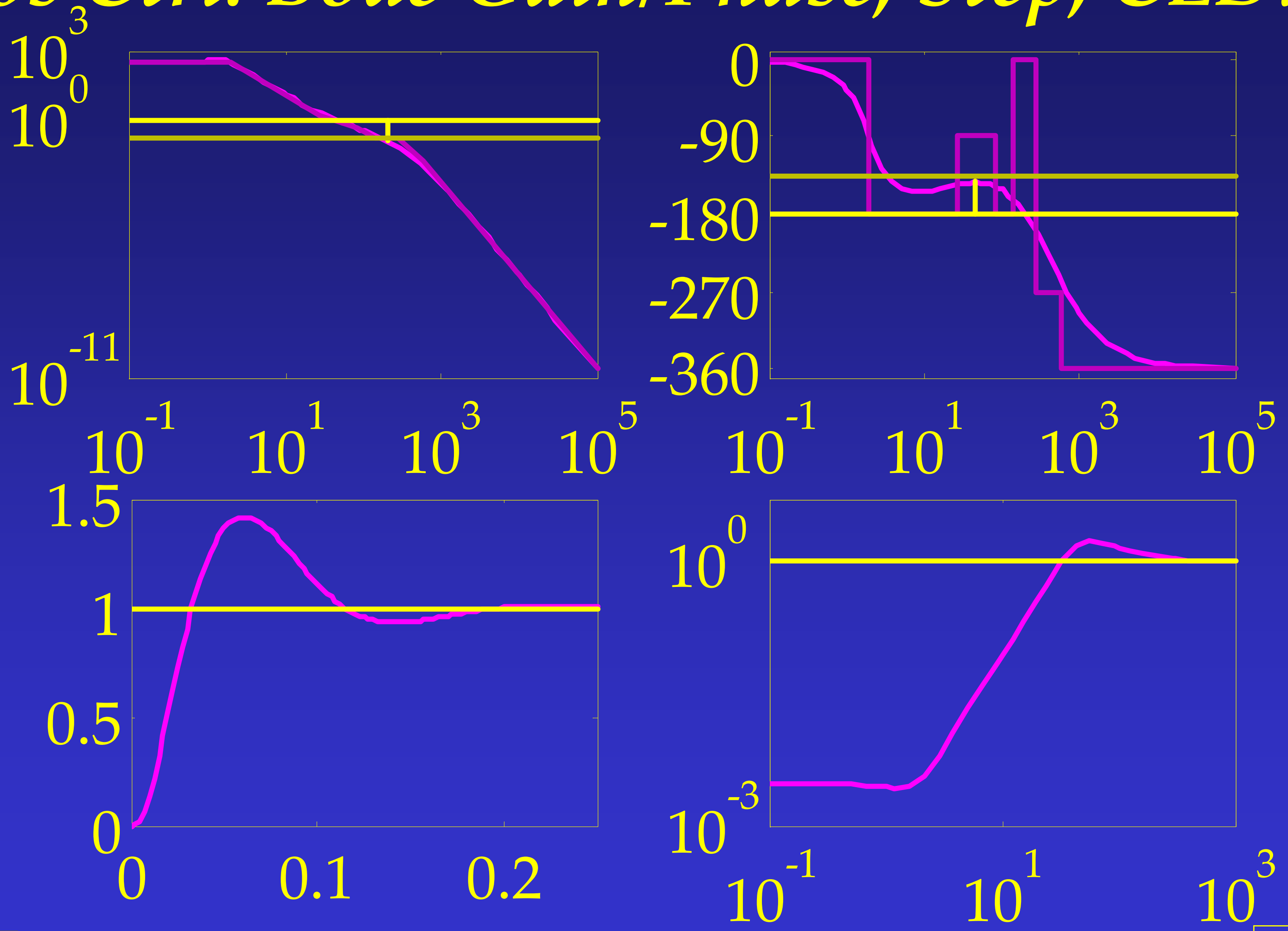
Do design, PM low (asymptotic approx), so redesign; %os tends to be high, so design again

Results

Also did Position Control (H(s) extra 1/s term) [d..f], and Computer Control hence with time delay [g])

	GMax	GM	PM	ω_{rf}	T_{pk}	O_{ss}	%os	Tset
a	330.7	17.8	36.8	37.12	0.061	0.997	42.1	0.23
b	263.4	15.0	44.6	43.22	0.053	0.996	31.1	0.18
c	168.8	15.4	53.1	32.21	0.055	0.994	20.3	0.25
d	346.7	15.6	36.9	43.54	0.058	1.000	41.9	0.18
e	249.7	15.7	45.0	31.75	0.060	1.000	31.1	0.14
f	189.8	12.6	52.1	32.06	0.048	1.000	19.5	0.24
g	110.0	15.6	56.7	19.79	0.093	0.991	20.2	0.26

Pos Ctrl: Bode Gain/Phase; Step, CLDist



Conclusion

Maximum Available Feedback is a good example of a non trivial design method

Although developed for Electronic Amplifiers, have shown how it can be applied for Control

It thus could fit into a Control Engineering syllabus

Other work: better ways of achieving PM (adjusting asymptotes), and selecting number of LeadLags: can exceed Maximum Available Feedback!

Future Work : more detailed comparison with other design methods