OBTAINING MAXIMUM FEEDBACK AND DESIRED PHASE MARGIN

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Overview

- Bode's fundamental work uses asymptotes to allow a system to be stabilised having suitable gain and phase margin, and max possible gain over a given bandwidth

  (It's a method of placing poles/zeros)

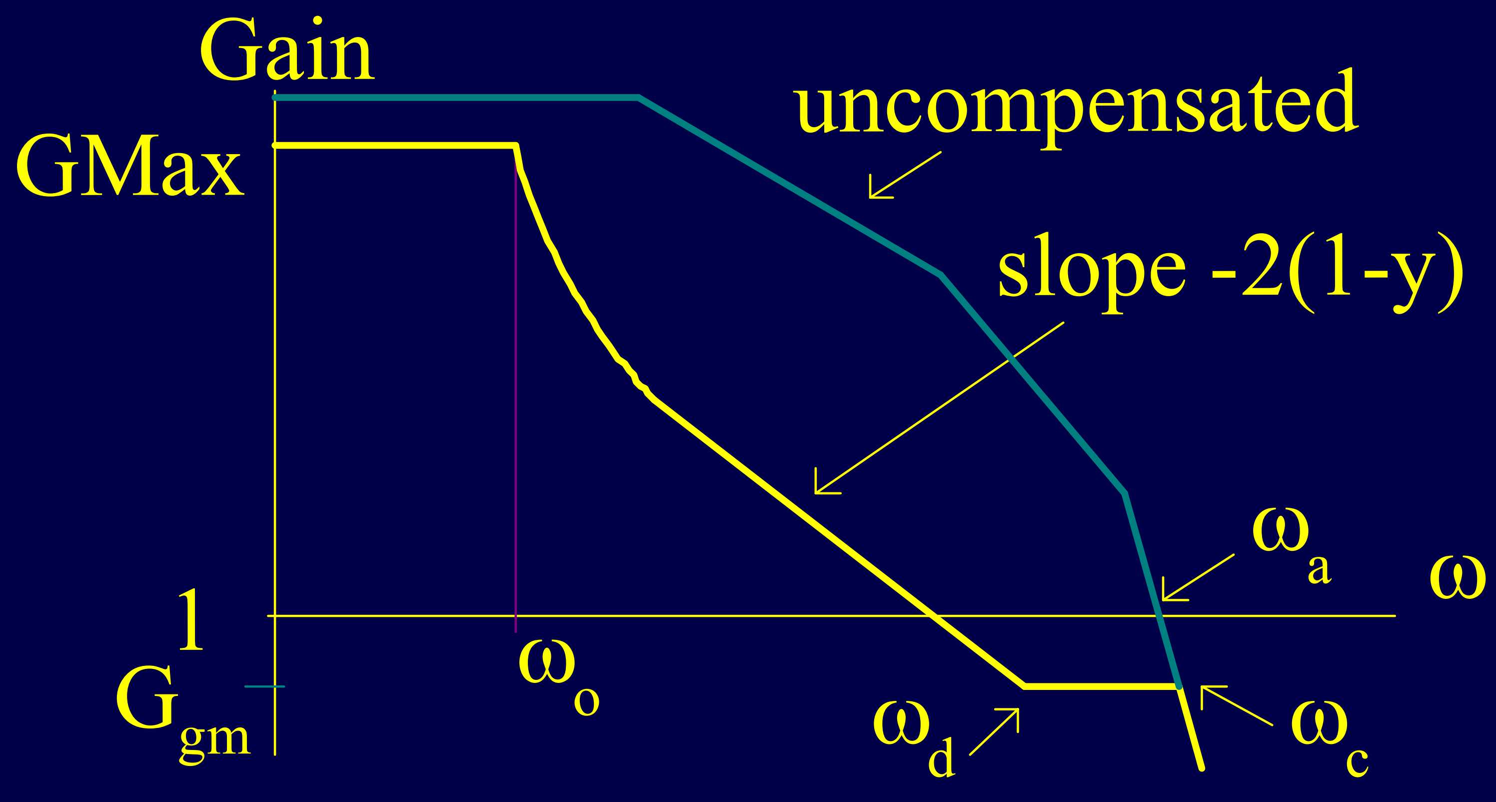
- But if specify too high a bandwidth, for instance, actual phase margin far too low
- A solution is presented, which is consistent with Bode's aims

Specification

- Uncompensated system
  - Gain = 1 at ω_a then its order is n
  - As Phase = -n*π/2, unstable if n>2
- Compensated system specified to have
  - Phase margin, PM
  - Gain margin, GM
  - Max possible gain up to ω_o (bandwidth)
- Define y = PM/π; x = GM, then

$$GMax = 40(1-y)\log_{10}\left(\frac{4(1-y)}{n}10^{x/20n}\frac{\omega_a}{\omega_o}\right) - x$$

Frequency Shape to achieve this



Slope -2(1-y) → Phase = -π + PM;

'Bode Step' ω_d .. ω_c: cancel phase due to –n slope

Loop Transfer Function

$$\frac{GMax}{T^2s^2+Ts+1}\frac{1+s/\omega_1}{1+s/\omega_2}\frac{\left(1+s/\omega_d\right)^2}{\left(1+s/\omega_c\right)^n}, \text{where } T=\frac{1}{\omega_o}$$

Second order element for low freq response.

(easier for students to understand than Bode's irrational element)

Lead Lag to approximate slope -2(1-y)

Can be better to have multiple lead lags

Problem

Slope -2(1-y) from ω_d/m to ω_d where

$$m = 2^{1-\frac{1}{y}} \frac{\omega_d}{\omega_o}$$

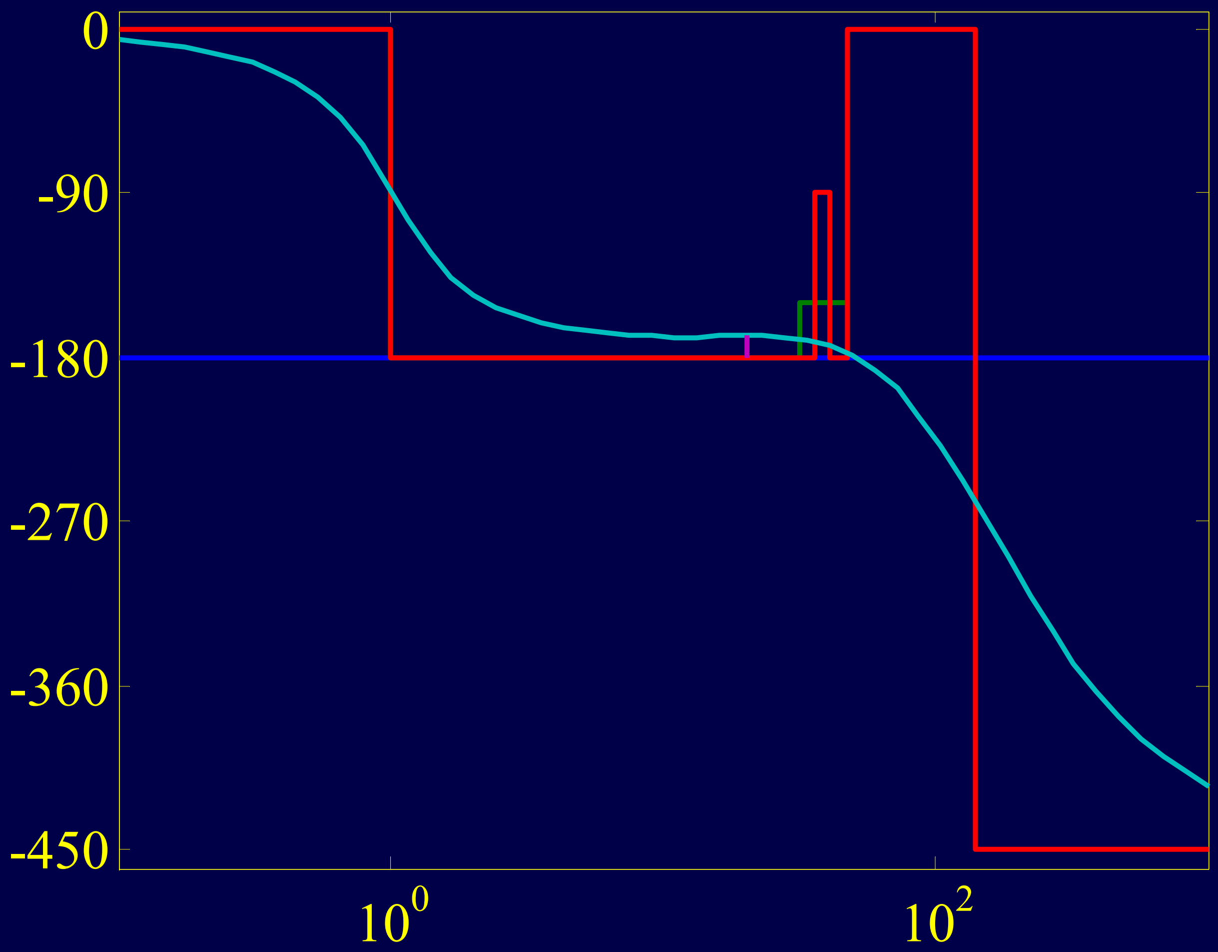
$$PM = \quad 30^O \qquad 45^O$$

$$2^{1-\frac{1}{y}} = 0.03 \qquad 0.125$$

For PM=30^O, ω_d must be at least 30 times ω_o and preferably much larger.

That is if bandwidth ω_o too large, there wont be region where slope -2(1-y) and phase not $-\pi$+PM

For instance – Phase response



ω_o 1rad/s

GM 15dB

PM 30^O

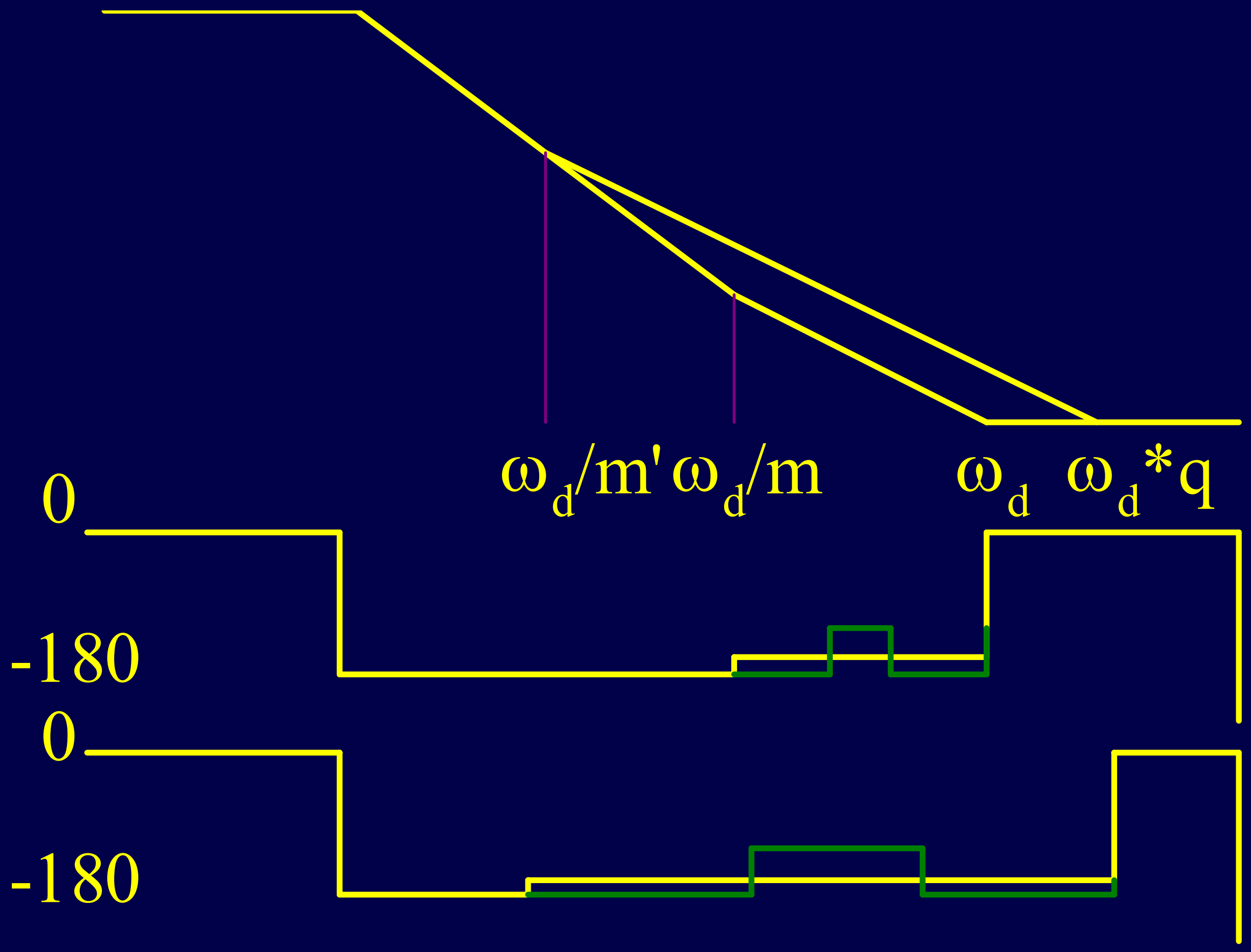
ω_a 100 rad/s

n 5

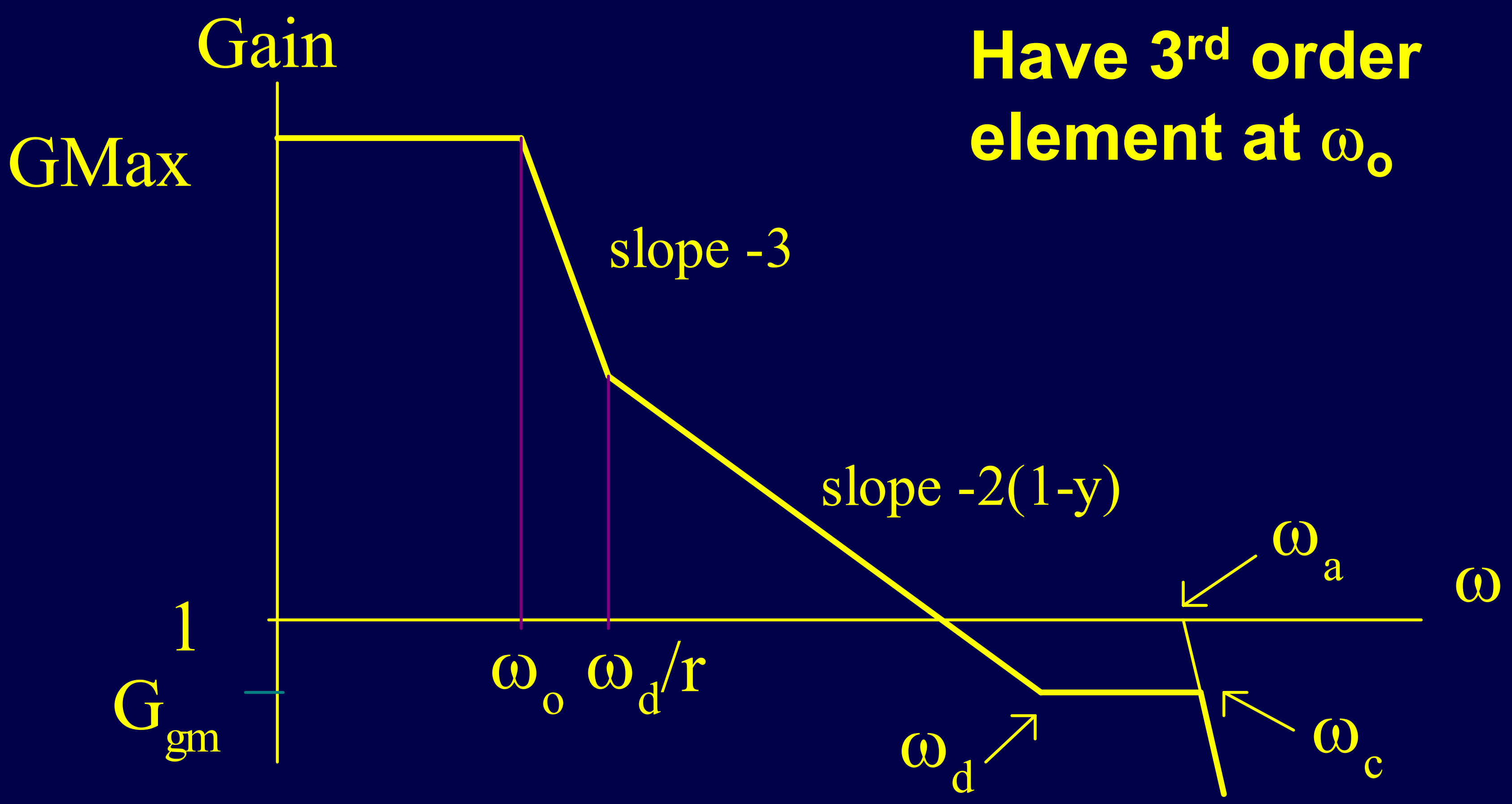
PM actual 11.6^O

Means of extending -2(1-y)

a) reduce Bode Step not good. ignores Bode's stability analysis



Better – extend -2(1-y) to low freq



Have 3rd order element at ω_o

Now 'length' of -2(1-y) given by

$$r = 2^{\frac{2(y-1)}{1+2y}} \frac{\omega_d}{\omega_o}$$

$$PM = \quad 30^O \quad 45^O$$

$$2^{\frac{2(y-1)}{1+2y}} = 0.42 \quad 0.5$$

For same system as shown earlier

PM actual was 11.6^O, with fix PM actual 27.5^O

IF GM reduced from 15dB to 10dB

PM actual 9.8^O or with fix PM actual 23.1^O

Can achieve PM=30^O if specify higher PM

Conclusion

- Analysis has shown why a design using Bode's method may not have the desired phase margin, particularly when seeking too high a bandwidth. For such situations, however, a simple successful solution to the problem is provided.

Acknowledgement

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