BODE'S MAXIMUM AVAILABLE FEEDBACK AND PHASE MARGIN

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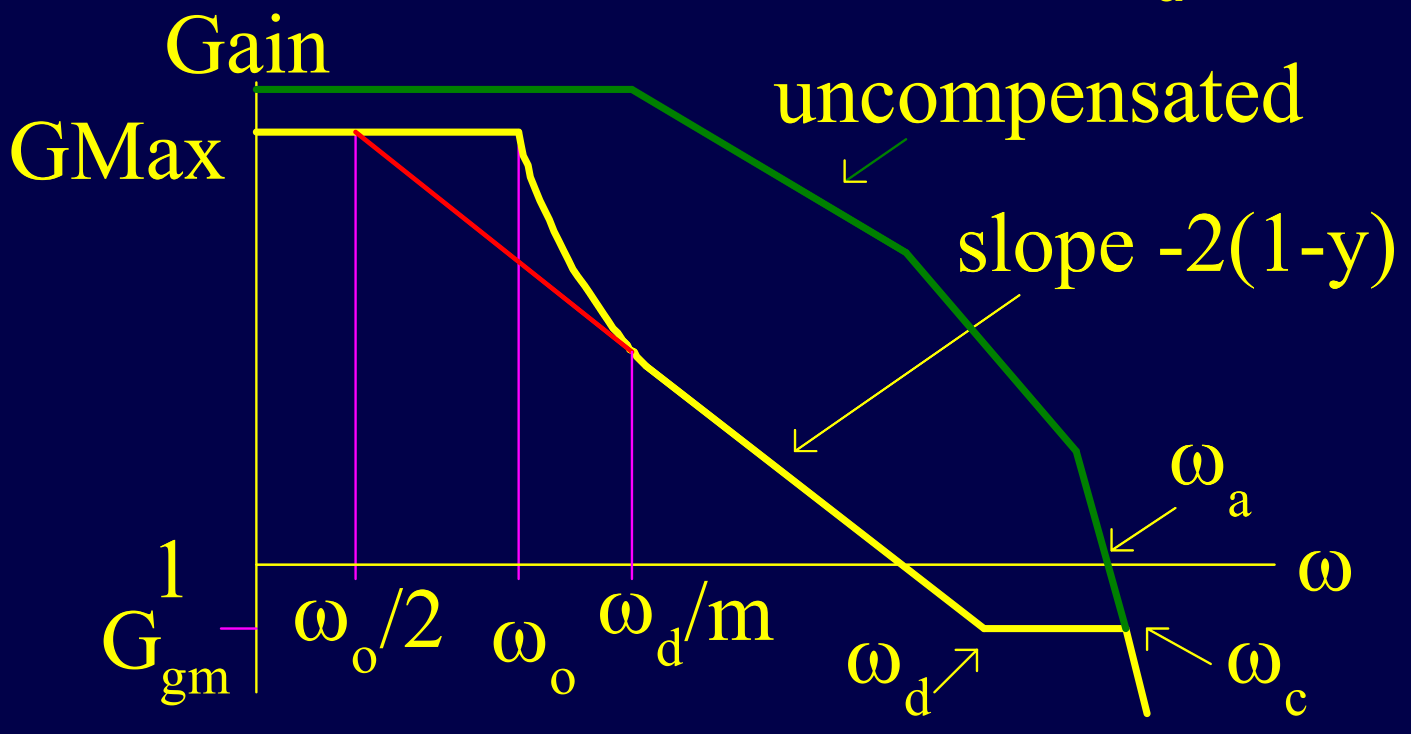
Overview

- Maximum Available Feedback is max loop gain over a specified bandwidth for given margins, in a single loop feedback system
- Uses asymptotes, so actual margins can be very different from specified – often phase margin is low
- In ASM2003 author showed how asymptotes can be changed for large bandwidth
- This paper considers further adaptions and how can be applied to smaller bandwidths

Frequency Shape for Bode's Design

Uncompensated: gain = 1 at ω_a when slope -n



Specify
ω_o = bw
x = Gain Margin
y = Rel Phase Margin PM/180

Slope -2(1-y) → Phase = -180 + PM;
'Bode Step' ω_d..ω_c: cancel phase due to –n slope

Loop Transfer Function – 3 parts

Design produces transfer function round loop

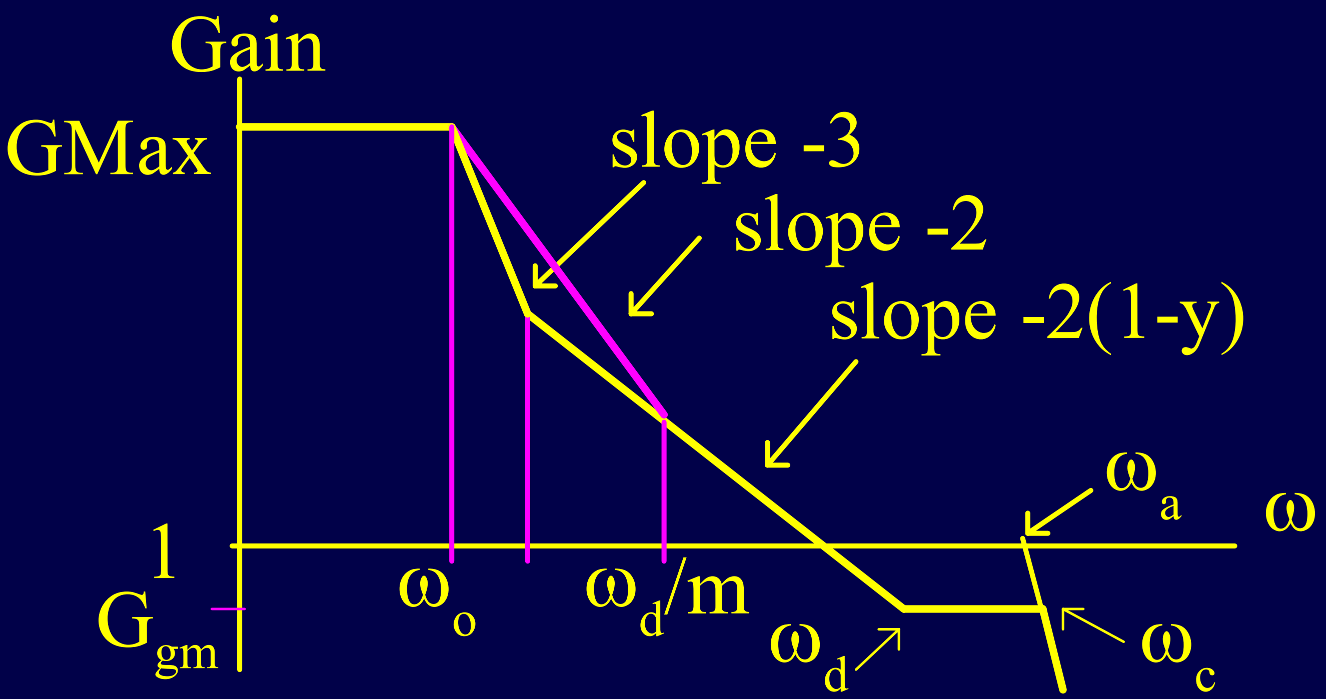
- Curved Part : low freq response
  - Bode's irrational element awkward, so
  - Second Order Element, corner freq ω_o
  - In effect slope -2 from ω_o to -2(1-y) slope
- Lead Lag(s) to approximate slope -2(1-y)
  - from ω_d / *m* to Bode Step (at ω_d)
- Double Lead for Bode Step at ω_d
- Then n Lags at ω_c

But Slope Can Be Too Short

$$m = 2^{1-\frac{1}{y}} \frac{\omega_d}{\omega_o}$$

$PM = 30^O \quad 45^O$

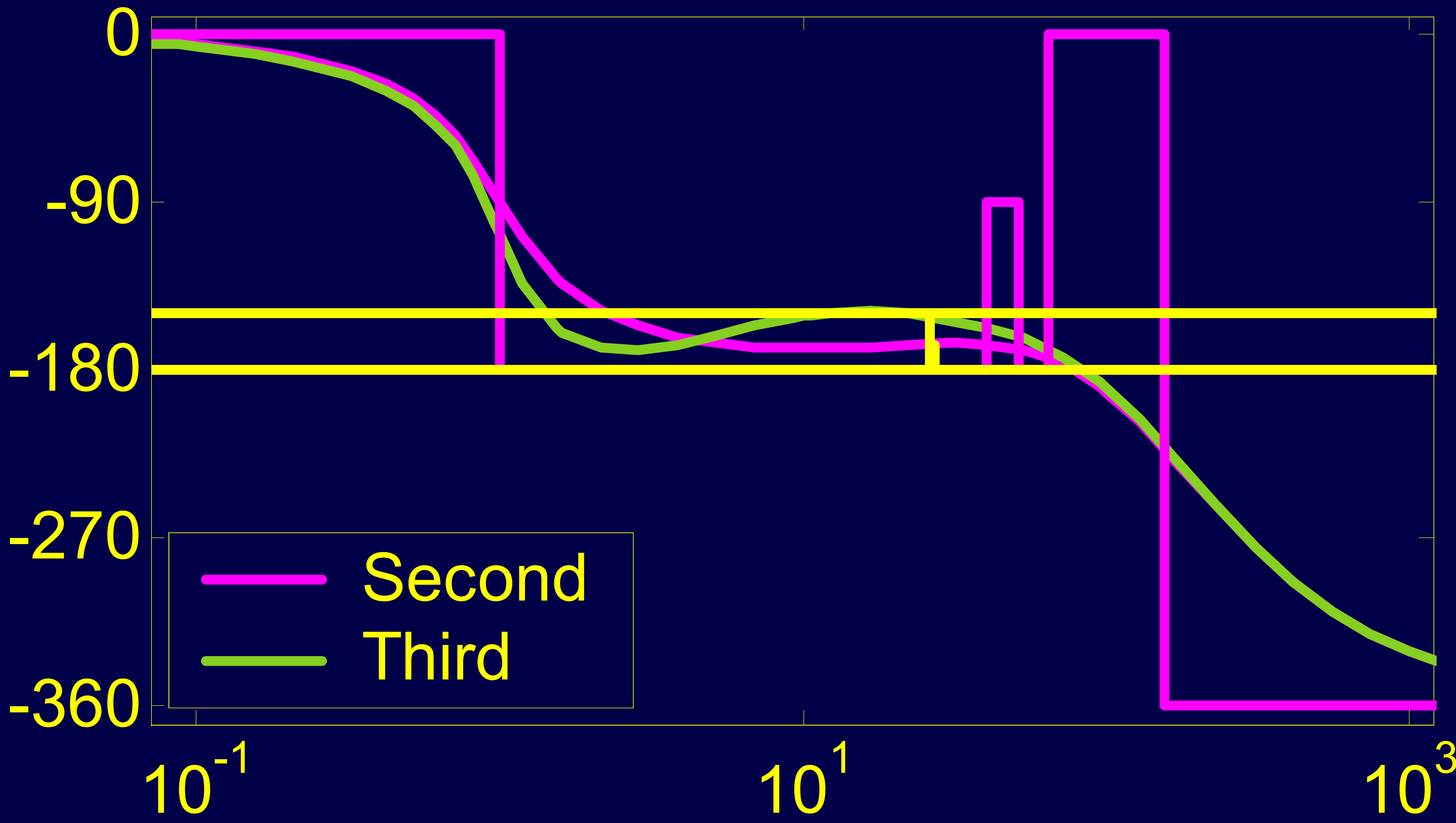
$2^{1-\frac{1}{y}} = 0.03 \quad 0.125$



So have third order element at ω_o, lead at ω_d/m

$$m' = 2^{\frac{2(y-1)}{1+2y}} \frac{\omega_d}{\omega_o}$$

Bode Phase Plot : Phase vs log(ω)



GM 15dB

PM 30^O

$\omega_o = 1$ rad/s

$\omega_a = 100$ rad/s

n = 5

Actual PM up from 13.9^O to 28.5^O

However

- As Phase lag goes past -180^{O} + PM soon after ω_{o}, does not meet Bode's PM defn:
  - If add phase lag, system conditional stable
- Thus investigated different configurations for region up to $\omega_e = (\omega_d / m)$
- Already 2nd and 3rd order elements (a) (b)
- Tried 3rd order at ω_o, lead mid way $\omega_o : \omega_e$
  - (in effect slope -3 then -2) (c)
- Also slopes -3, -2 then -1 (d)
- Also slopes -3 then -1 (e) and -4 (f)

Example Results

	PM spec = 30°		PM spec = 45°		PM spec = 60°	
Sys	PM	MaxPh	PM	MaxPh	PM	MaxPh
a	13.9	-169	38.7	-153	58.0	-136
b	28.5	-169	45.6	-153	61.4	-135
c	28.6	-170	45.5	-155	61.0	-139
d	37.1	-181	45.5	-156	60.0	-140
e	41.0	-185	46.4	-156	60.3	-139
f	26.7	-168	43.7	-149	59.3	-132

Max Phase means still not meet Bode's PM defn

d) & e) not good as can be conditionally stable

Step Response Tests

	PM spec = 30° GMax = 588			PM spec = 45° GMax = 223			PM spec = 60° GMax = 86		
Sys	Tpk	%os	Tset	Tpk	%os	Tset	Tpk	%os	Tset
a	0.11	76.0	1.25	0.15	39.6	0.59	0.21	20.5	0.53
b	0.12	53.5	0.54	0.14	30.3	0.51	0.21	16.3	0.75
c	0.12	54.3	0.56	0.14	31.2	0.35	0.21	17.3	0.71
d	0.15	44.2	0.49	0.16	31.7	0.43	0.22	19.0	0.63
e	0.15	40.1	0.59	0.16	30.5	0.35	0.22	18.6	0.65
f	0.11	56.0	0.53	0.13	32.6	0.52	0.20	18.6	0.72

No obvious best

PM = 45^o; different ω_o and LeadLags

Sys	ω_o=1; LL=1			ω_o=0.1; LL=1			ω_o=0.1; LL=2		
	PM	Tpk	Tset	PM	Tpk	Tset	PM	Tpk	Tset
a	38.7	0.15	0.59	45.2	0.13	0.84	45.6	0.15	0.62
b	45.6	0.14	0.51	37.6	0.12	0.77	48.0	0.15	0.39
c	45.5	0.14	0.35	39.7	0.12	0.92	47.6	0.15	0.35
f	43.7	0.13	0.52	35.9	0.12	0.72	48.0	0.15	0.52

For ω_o = 0.01, get similar good result if LL=3

Paper has similar results for PM = 30^o and 60^o

Overall, configuration c) seems best

Conclusion

- Modifying the linear element used for the low frequency response, and choosing the appropriate number of lead-lag elements for the -2(1-y) slope successfully ensures Maximum Available Feedback and Phase Margin are achieved
- Worth trying different configurations
- An automatic method of selecting lead-lags is needed … the author is working on one!