Experiment 6: STUDY OF A POSITION CONTROL SERVOMECHANISM

1. Objectives

This experiment provides the student with hands-on experience of the operation of a small servomechanism. This system will be used for more complex work later in the course. The parameters of the system will be evaluated by a series of tests. The setpoint following operation of the servo will be studied qualitatively for different values of forward-path gain, and quantitatively for one value of gain. Theoretical predictions will be compared with measured responses.

2. Introduction

The hardware of a basic position control servomechanism is provided. It consists essentially of a potentiometer-pair error detector, a preamplifier, a circuit capable of connection as a series compensator, a voltage-to-voltage power amplifier, a DC motor with integral tachogenerator, a gearbox, and a load. The actual arrangement is more complicated than the front panel picture (Fig. 1) would suggest. In the arrangement provided for this experiment the effects of motor armature inductance and non-linear friction have been masked over the working range of the basic system. This has been achieved by the use of high gain techniques in association with internal feedback. The resultant system has been 'slowed down' electronically so that its performance over the range of the given experimental work is essentially linear and second order when switched to System 1 and third order when switched to System 2. Outside the range of this work (e.g. when the forward path gain is increased well beyond the values required) the system does begin to exhibit behaviour that cannot be explained on the basis of second order transfer functions. This need not concern the students performing this experiment, but these effects will be studied in later experiments.

With reference to Fig. 1, the position of a rotatable mass (also called the output position $\theta_o$) is to be controlled in accordance with the position of a command handwheel (also
called the input position $\theta_i$). The output is transduced to form a voltage $V_o$ and the input is transduced to form a voltage $V_i$. These are subtracted electronically using operational amplifiers to form an error voltage $V_e = P E_r$ where $E_r$ is the mechanical error given by $E_r = \theta_i - \theta_s$. The system may be isolated from mechanical commands by means of switch $S_1$. There is an auxiliary input $V_x$ for direct electrical commands that can be used for certain tests. The signal $V_e$ will normally be fed via switch $S_2$ to form $V_e'$. The net control signal $V_e'$ is then passed through an adjustable potentiometer of setting $A$ and a pre-amplifier before application to the series compensator. With switches 3 and 4 closed this compensator is a simple amplifier with gain 3. The terminals provided across these switches allow passive components (i.e. resistors and capacitors) to be connected in such a way as to produce phase-lead compensation, phase-lag compensation, proportional plus integral control P+I, or P+I+ phase lead compensation.

The output of the compensator is then combined with the local velocity feedback signal before being power-amplified and fed to the motor. The shaft of the motor is coupled via a 5:1 step down gearbox (employing a toothed belt for quietness) to the rotatable mass whose position is to be controlled. The major loop may be opened by means of switch $S_6$ and the auxiliary velocity feedback loop may be opened by means of switch $S_5$. The tachogenerator is mounted on the motor shaft. Its voltage is amplified and some of the commutator ripple is filtered before feeding to the terminal as $V_t$. The proportion of $V_t$ feedback can be controlled by the ten-turn potentiometer of coefficient $B$.

An analysis of the linearised system reveals that when switched to System 1, the system can be represented by the transfer function model illustrated by the block diagram shown in Fig. 2. Useful equations are developed in Appendix 1.
3. **Determination of system parameters**

Switch to System 1, and leave switches $S_3$ and $S_4$ closed for this experiment.

### 3.1 Potentiometer Responsivity $P$ (V/rad)

Close switch $S_1$, and open $S_2, S_5, S_6$. Then $V_e (volts) = P \theta_i$.

Record this voltage on AVO on 10 V range with $\theta_i$ set to (say) $0^\circ, \pm 45^\circ, \pm 90^\circ, \pm 135^\circ$. Plot a graph and hence evaluate $P$ (with expected tolerance). Confirm by recording $V_e$ with $\theta_i$ set to $1 \text{ rad } = 57.3^\circ$.

### 3.2 TachometerResponsivity $G$ (V/rad s$^{-1}$)

Close switch $S_2$. Set potentiometer $A$ to 0.40, vary $\theta_i$ and record $V_t$ and time for $n$ revolutions of the output shaft. Hence evaluate $W_o$ (in rad/s) and plot a graph of $V_t$ against $W_o$ to determine $G$.

*HINT.* Tabulate $V_t$, $n$, time for $n$ revs, $W_o$. Think about the accuracy of your readings.

In the future, $V_t$ will be used to determine shaft speed.

### 3.3 Amplifier/Motor/Load Parameters $M$ (rad s$^{-1}$ / V), $T$ (s)

The steady state relationship between $V_t$ and $\theta_i$ is given by:

$$V_t = P3AMG \theta_i$$

Record suitable values, plot a graph and hence evaluate $M$. 

3
The transient response of \( V_t \) to changes in \( V_x \) is of the form:

\[
V_t(t) = V_{t0}(1 - e^{-t/T})
\]

where \( V_{t0} \) is the steady state value and \( T \) is the time constant.

You will carry out a step test using VISSIM to record the open loop transient response. Double click on the icon labeled \textbf{Exp06}, which starts VISSIM and loads a suitable configuration for this experiment. Using the mouse, connect the icon labeled \textbf{Step 0 to 4} to the icon labeled \( V_x \). Connect the icon labeled \textbf{Step 0 to 4} to the plot window and connect the icon labeled \( V_t \) to the plot window. Then, carry out the physical connections shown in Table 1.

<table>
<thead>
<tr>
<th>Terminal on servo</th>
<th>Signal Number on box</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_x )</td>
<td>20</td>
</tr>
<tr>
<td>( V_0 )</td>
<td>36</td>
</tr>
<tr>
<td>( V_t )</td>
<td>37</td>
</tr>
<tr>
<td>Earth</td>
<td>11</td>
</tr>
<tr>
<td>Earth</td>
<td>18</td>
</tr>
<tr>
<td>Earth</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>24 to 5</td>
</tr>
</tbody>
</table>

\textbf{Table 1:} Physical connections
Ensure that the data acquisition channels are assigned in VISSIM according to Table 2.

<table>
<thead>
<tr>
<th>Type of conversion</th>
<th>Signal name</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>D/A</td>
<td>$V_x$</td>
<td>0</td>
</tr>
<tr>
<td>A/D</td>
<td>$V_0$</td>
<td>1</td>
</tr>
<tr>
<td>A/D</td>
<td>$V_1$</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 2**: Data acquisition channels

Use VISSIM (after an explanation by the demonstrator) to obtain a suitable transient response, and hence determine the time constant $T$. A step change in $V_x$ from 0 to 4 V with a value of $A = 0.40$ should be suitable.

If time is short, you may assume $T = 0.22$ s.

4. **Closed-loop tests on Positional Servo**

4.1 **Response with adjustable gain**

Set $A = 0.30$, close switches $S_1$ and $S_6$, open $S_5$, and examine how $\theta_o$ follows manual variations of $\theta_i$. Reduce $A$ towards zero (e.g. set A to 0.10, 0.025, 0.01), and then increase it towards 1.00 (e.g. set A to 0.50, 0.70, 1.00). At each setting of $A$, observe the following-action of the servo and comment on the effect of altering the forward-path gain on the servo performance.
4.2 Step Response with given parameters

Set $A = 1.00$ and open $S_1$. Use VISSIM to obtain recordings of $V_o$ and then $V_i$ for step changes of input voltage $V_x$ from -4 to 0. Determine the percentage overshoot, and time to peak, and compare with values predicted from the equations for the servo. (see Appendix 1). Confirm that $V_o$ is a maximum when $V_i$ crosses zero.

Appendix 1

With reference to Fig. 2, when $S_5$ is open and all others are closed (NOTE. With switch $S_5$ OPEN, the value of $B$ is effectively zero). The transfer function from $\theta_i$ to $\theta_o$ is given by:

\[
\frac{\theta_o(s)}{\theta_i(s)} = \frac{P3A \left[ \frac{M}{(1+sT)} \right] \frac{1}{s}}{1+3A \left[ \frac{M}{(1+sT)} \right] \frac{1}{s} P}
\]

This transfer function can be re-written as follows:

\[
\frac{\theta_o(s)}{\theta_i(s)} = \frac{3AMP}{T \left[ s^2 + \frac{1}{T} s + \frac{3AMP}{T} \right]}
\]

(A1)

Now consider the standard second order transfer function:

\[
G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

(A2)
Comparing equations (A1) with (A2) it is easy to show that the undamped natural frequency is given by:

\[ \omega_n = \sqrt{\frac{3AMP}{T}} \]  \hspace{1cm} (A3)

while the damping coefficient \( \zeta \) is given by:

\[ \zeta = \frac{1}{2T\omega_n} = \frac{1}{2T} \sqrt{\frac{T}{3AMP}} = \frac{1}{2\sqrt{3AMPT}} \]  \hspace{1cm} (A4)

Compute the values of the damping coefficient and undamped natural frequency for the servo system by using Eqns. (A3) and (A4).

The values of the percent overshoot \( M_p \) and time to peak \( t_p \) for a second order system like (A2) are given by:

\[ M_p = \frac{MaxValue - FinalValue}{FinalValue} \times 100 = \exp \left( \frac{-\zeta \pi}{\sqrt{1 - \zeta^2}} \right) \times 100 \]  \hspace{1cm} (A5)

\[ t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} \]  \hspace{1cm} (A6)

Calculate the predicted values of \( M_p \) and \( t_p \) using Eqns. (A5) and (A6).

Revised by V. Becerra, September 2000.