The Kalman filter

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The basic idea

- In the Kalman filter we assimilate the observations sequentially,making use of the equation we found in the first lecture.

$$\mathbf{x} = \mathbf{x}_b + \mathbf{P}\mathbf{H}^T(\mathbf{H}\mathbf{P}\mathbf{H}^T + \mathbf{R})^{-1}(\mathbf{y} - H(\mathbf{x}_b))$$

- The background state comes from the forecast of the previous analysis.
- In the Kalman filter, the uncertainty on the background comes from a forecast of the uncertainty on the analysis.

Framework

- We assume a linear model and observation operator.
- The model may be imperfect.

$$\mathbf{x}_{i+1}^{t} = \mathbf{M}_i \mathbf{x}_i^{t} + \boldsymbol{\eta}_i$$

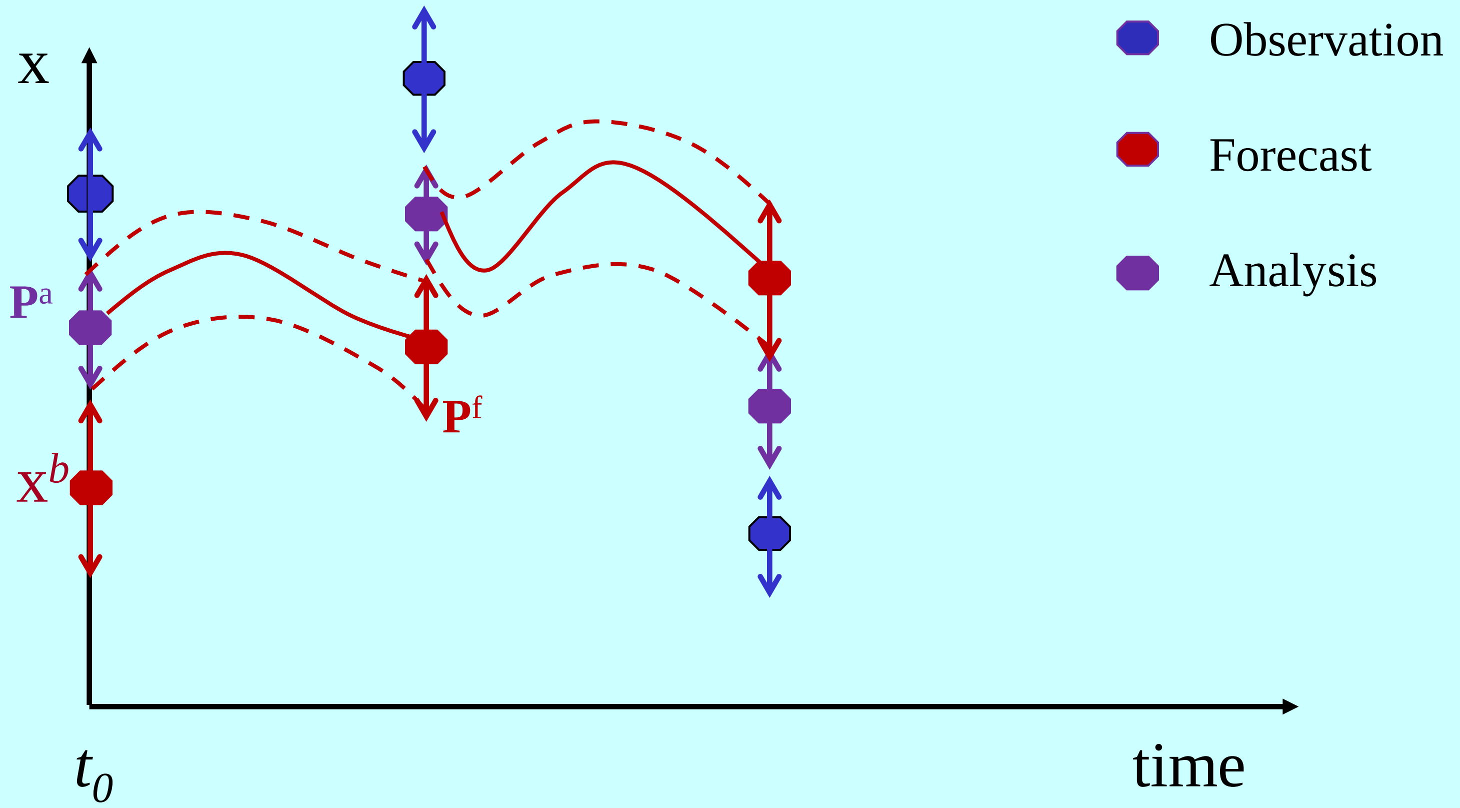
with

$$< \boldsymbol{\eta}_i > = 0$$

and

$$< \boldsymbol{\eta}_i \boldsymbol{\eta}_i^T > = \mathbf{Q}_i$$

Kalman filter - Illustration



We have the following steps:

- Kalman gain computation

$$\mathbf{K}_i = \mathbf{P}_i^f \mathbf{H}_i^T (\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R}_i)^{-1}$$

- State analysis

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K}_i (\mathbf{y}_i - \mathbf{H}_i \mathbf{x}_i^f)$$

- Error covariance of analysis

$$\mathbf{P}_i^a = (\mathbf{I} - \mathbf{K}_i \mathbf{H}_i) \mathbf{P}_i^f$$

- State forecast

$$\mathbf{x}_i^f = \mathbf{M}_{i-1}\mathbf{x}_{i-1}^a$$

- Error covariance forecast

$$\mathbf{P}_i^f = \mathbf{M}_{i-1}\mathbf{P}_{i-1}^a\mathbf{M}_{i-1}^T + \mathbf{Q}_{i-1}$$

Notes

- Under the assumptions (including linearity) the Kalman filter is the optimal way to assimilate observations sequentially.
- In practice the scheme is too expensive for large systems, but provides a useful reference for the design of ensemble methods.
- For a linear system, with no model error, the analysis at the end of the time window is the same as the final 4D-Var analysis using the same data.

Example

See separate sheet