Final Report on EPSRC Grant GR/M59433/01
INTEGRAL EQUATION METHODS FOR DIRECT AND INVERSE SCATTERING BY UNBOUNDED SURFACES

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1. Background/Context

This project has been concerned with the development and analysis of numerical algorithms, based on integral equation formulations, for scattering by unbounded surfaces, in particular what are termed *rough surface scattering problems* in the engineering literature. (The phrase *rough surface* is used in the engineering literature to denote surfaces which are a (usually non-local) perturbation of an infinite plane surface, the perturbation such that the whole surface lies within a finite distance of the original plane.)

The motivation for the study of these problems comes from a range of applications. Such problems arise frequently in applications, in particular in modelling acoustic and electromagnetic wave propagation over outdoor ground and sea surfaces, and are the subject of intensive studies in the engineering literature, with a view to developing both exact methods of computation and approximate, asymptotic, or statistical methods. For recent surveys of available methods and many practical applications see Voronovich [9], Warnick & Chew [10], Saillard & Sentenac [8], De Santo [4].

Within the UK the engineering interest in surface scattering problems led to an Institute of Acoustics conference on Acoustic Characteristics of Surfaces: Measurement, Prediction and Applications in September last year. This meeting considered novel prediction methods (with the results of this research project represented by three papers), and applications including: deducing surface shape and properties from indirect acoustic measurements; noise propagation over rough ground surfaces; seismic scattering from the sea floor; and the design of wall and ceiling profiles in concert acoustics [2]. In the electromagnetics community the large interest in rough surface scattering problems is illustrated by the presence of 4 minisymposia and over 50 papers, dealing with new prediction algorithms, the inverse problem, and a variety of applications, at the 2002 Progress in Electromagnetics Symposium, Boston [1].

Reflecting the theme of this research project, the most popular methods in the engineering literature for exact computation of rough surface scattering continue to be boundary integral equation methods, which are used in their own right for accurate predictions, and to validate other, approximate methods [10, 8, 6].

Thus this research project has taken place in the context of continued, intense and enthusiastic interest in modelling direct and inverse problems of surface scattering using boundary integral equation methods. The project has also taken place in the context of increasing mathematical international interest in direct and inverse wave scattering and their numerical computation (see e.g. the many conferences cited in dissemination). As the project has unfolded there has also been increasing interest in operators on unbounded domains with difficult limiting behaviour at infinity (see e.g. the contributions of Silbermann, Lindner, Rabinovich, and Roch in [5]), as exemplified by boundary integral operators on unbounded surfaces which are the graph of functions with no simple asymptotic limit at infinity.

2. Objectives, Project Plan Review, Research Staff: Their Outputs and Training

Objectives. The objectives of the research as specified in the original application were:

1. To develop theories of the spectral and Fredholm properties of integral operators on unbounded domains relevant to integral equations arising in rough surface scattering problems and well-posed integral equation formulations for three-dimensional rough surface scattering problems.

2. To develop numerical methods for integral equations on unbounded surfaces, in particular the boundary integral equations arising from rough surface scattering problems, stability and convergence analyses for these methods, analysis of effects of 'finite section' errors (truncating the domain of integration), and efficient matrix-vector multiply and iterative solution schemes.

3. To develop practically useful algorithms and software, based on boundary integral equation formulations and the results of 1 and 2, for two- and three-dimensional boundary value problems for the Helmholtz equation and time harmonic Maxwell's equations, in particular problems of scattering by rough surfaces and interfaces and problems of scattering by obstacles buried below such interfaces.

4. To develop and analyse algorithms and software for problems of inverse scattering by unbounded surfaces and by obstacles (bounded and unbounded) buried below such surfaces.

Project Plan Review. To achieve these objectives, the original grant application requested a 3-year Postdoc and a 3-year project studentship. In the original proposal it was proposed that the PhD student be responsible particularly for objective 4. In the event, the 3-year project studentship part requested **was not funded**. However, due to contributions to the project of other research students and visitors, the adverse impact of this on achievement of the objectives was limited. However, the shortfall in funding did mean that part of the effort of the Postdoc was diverted from objective 3 to objective 4, so that progress on objective 3 was not as marked as we would have liked, though it still led to significant software which has been used in industry.

Additionally, opportunities arose, during the course of the research, to explore research areas not envisaged in the original proposal. In particular, arising out of research on domain derivatives and Newton methods for the rough surface problem, new analysis methods and connections with other algorithms became apparent for Newton's method for scattering by bounded obstacles, and some effort was expended in work in this direction [J1]. Similarly, arising from work on an efficient algorithm for a surface scattering problem [J11], the possibility of using the same techniques for efficiently computing scattering by bounded obstacles became apparent and was explored in the last section of [J11] and in other work in preparation.

Research Staff. The project experienced the usual challenges in attracting and retaining good Postdoc staff in the face of the attractions of higher salaries in other sectors and/or the prospect of more permanent appointments. We were very fortunate, however, in attracting, over the course of the grant, three very good Postdocs with very relevant experience. These staff were, in chronological order:

Dr Roland Potthast, a very experienced researcher, with the German Habilitation, highly respected in the inverse problems and scattering community, who joined us for the first year of the project following a year in industry. After one year, sadly for us, he was head-hunted to return to Goettingen to head a Young Researchers Group on 'New Numerical Methods for Inverse Problems' on a longer-term contract. We did, however, maintain close contact and collaboration, to work on publications and to jointly organise a minisymposium on 'Direct and Inverse Surface Scattering' at the JEE '02 European Symposium on Electromagnetics, Toulouse 2002.

Dr Steve Langdon, who joined us for 15 months following a PhD on boundary integral equations methods with Ivan Graham and a Postdoc with John Barrett and James Blowey on finite element methods for degenerate parabolic problems. Steve left to take up a further Postdoc position with Ian Sloan and Mahadevan Ganesh at the University of New South Wales. He will return to the UK to take up a Leverhulme Early Career Fellowship on 'Numerical Simulation of High Frequency Scattering' followed by a Lecturer appointment, at Reading in May this year.

Dr Claire Lines, who participated throughout the project, supported by an EPSRC CASE Studentship with the Radio Communications Unit at Rutherford Appleton Laboratory and, following her studentship, was employed for three months using the last of the Staff Funding on the project. She is currently writing up further results from the project and working as a lecturer at Brunel.

Both Roland Potthast and Steve Langdon were appointed, in view of their extensive relevant experience, at scale points significantly higher than envisaged in the original application, so that the staff budget ultimately funded only 30 rather than 36 man-months. The experience of the research staff compensated, however, amply for the reduction in total man-months.

As mentioned above, the project also benefitted from contributions of students and visitors, including:

Anja Meier, EPSRC CASE Student with the Transport Research Laboratory Limited. PhD on 'Numerical treatment of integral equations on the real line with application to acoustic scattering by unbounded rough surfaces' awarded May 2001.

Claire Lines, EPSRC CASE Student with the Radio Communications Unit, Rutherford Appleton Laboratory. PhD on 'Inverse scattering by unbounded rough surfaces' awarded July 2003.

Dr Tilo Arens, EPSRC Mathematics Postdoctoral Fellow at Brunel for one year, 2000-2001.

Professor David Natroshvili, Georgian Technical University, Tbilisi, visitor for 12 weeks in the Autumn of 2000, funded by the Royal Society.

Professor John DeSanto, Colorado School of Mines, visitor for two months in Spring 2001.

Other significant collaborators in the course of the project were **Dr Bo Zhang**, Reader in Applied Mathematics, Coventry University and **Kai Haseloh**, a PhD student at the University of Hannover.

Training Opportunities. The training opportunities provided to the research staff were outstanding. All the research staff had opportunities to interact significantly as part of a larger team working on numerical methods for direct and inverse scattering problems (which comprised Postdocs Potthast and Arens, PhD students Rahman, Meier, Lines, Chunrungisikul, Birbiad, and long-term visitors Natroshvili and/or DeSanto for a large part of the grant period). All were encouraged and funded to attend key international meetings and present and obtain feedback on their results and make appropriate visits to groups overseas (e.g. Langdon visited Elschner's group working on direct and inverse problems in diffractive optics at the WIAS, Berlin for several weeks and the *Projet Ondes* group of Patrick Joly at INRIA, Versailles; Lines visited and gave seminars in Göttingen and Karlsruhe and presented 10 papers/posters at national and international conferences). Langdon and Lines were encouraged and supported in applying for individual fellowships to further their independent research careers - Langdon has obtained a Leverhulme Fellowship while Lines' application is pending. Lines and Potthast had significant industrial interaction with Rutherford Appleton Laboratory and the Transport Research Laboratory Limited, respectively, Potthast involved in a short-term consultancy project for the Noise and Vibration Unit at TRL. All the research staff took the opportunity, within the limits specified by EPSRC and with appropriate training, to gain skills in lecturing in areas relevant to their research activity (e.g. numerical analysis, finite element methods).

Research Staff Outputs. The combined outputs from the research staff on the project comprise one book, three refereed journal papers published, two further journal papers submitted, and 8 papers in conference proceedings (see the Appendix). At least three further refereed journal papers are in preparation. Additionally, all staff were involved in producing significant software.

3. Key Advances and Project Outputs

The project has achieved many very significant advances in the mathematical and numerical analysis of integral equation methods for direct and inverse scattering. The outputs from the project (see the Appendix and section 5 below) include 11 refereed journal papers, with 3 others submitted and others in preparation, 11 papers in conference proceedings, and software products licensed to the Transport Research Laboratory Limited. Key areas of development are listed below, ordered to follow as far as possible the list of objectives above:

1. The development of theories of the spectral and Fredholm properties of integral operators on unbounded domains relevant to integral equations arising in rough surface scattering problems. Under this heading:

(i) We have developed a general theory of the solvability of second kind operator equations on Banach spaces (a generalised collectively compact operator theory [J4]), introducing new topologies on a Banach space, weaker than the norm topology, and studying operators of the form $I + K$, where I is the identity operator and K, with respect to the weaker topology, is continuous and maps bounded sets to precompact sets (is *Montel* in the terminology of topological vector spaces). In the particular case in which Ω is some subset of $\mathbb{R}^n$ and the Banach space is $BC(\bar{\Omega})$, the set of bounded continuous functions on $\bar{\Omega}$, the weaker topology is precisely the strict topology of Buck [3]. (The theory is much more generally applicable however, for example to the sequence spaces $\ell^p(\mathbb{Z}^n)$, in which every bounded operator is Montel.) The theory provides: (a) given a set S of bounded linear operators, criteria on S which ensure that if $I + K$ is injective for all $K \in S$ then $I + K$ is invertible for all $K \in S$, with the inverses uniformly bounded for $K \in S$; (b) criteria on a sequence $(I + K_n)$, approximating an operator $I + K$, which ensure that if K_n converges to K strongly (i.e. pointwise) with respect to the weaker topology then the approximating sequence is stable and strongly convergent (i.e., for some N, $\sup_{n\geq N} \|(I + K)^{-1}\| < \infty$ and $(I + K_n)^{-1}$ converges strongly to $(I + K)^{-1}$). The theory is applied to obtain results on the solvability of general systems of weakly singular second kind integral equations on unbounded domains in [J4]; these results are applied in turn to show that specific integral equation formulations of rough surface scattering are well-posed in [J10]. The results of [J4] are applied directly to establish the stability and convergence of finite section schemes in [J2].

(ii) We have studied the leading order asymptotics at infinity of solutions of integral equations on unbounded domains. This is of interest for a number of reasons: the decay rate of wave fields at infinity is obviously of intrinsic importance; the faster the decay rate the more accurate finite section methods are; the faster the solution decays the faster numerical solutions should decay, to maintain small relative errors at infinity (which are usually required in scattering problems). This question requires a study of the solvability of the integral equation in weighted spaces of continuous functions [J5,J12]. It turns out that this provides a key tool to establish solvability in L^p spaces $1 \leq p \leq \infty$ [J8].

2. The development of well-posed integral equation formulations for two- and three-dimensional rough surface scattering problems. Following on from earlier work on the 2D Dirichlet rough surface scattering problem (reviewed in [J6]), rigorous uniqueness and existence results and well-posed boundary integral equation formulations have been obtained for 2D scattering by a rough surface with (possibly inhomogeneous) impedance boundary condition [J7] and, using results from [J9], for the case of a rough interface between acoustic media with different wave speeds and densities [J10]. (The impedance boundary value problem is that solved by the software supplied to TRL, discussed below.) Using the results of [J5,J8] we can show that these integral equation formulations are well-posed in the whole scale of L^p spaces and in weighted spaces of continuous functions, so that sharp bounds on decay at infinity of the wave field can be obtained [J8]. Progress for the corresponding 3D problems has been limited, in part for the reasons discussed in section 2 and in part because progress proved more difficult than anticipated. A paper is in progress and will shortly be submitted to *SIAM J. Appl. Math* which deals with the case where the surface is the graph of a function $f \in C^{1,1}(\mathbb{R}^2)$ which has sufficiently small norm. Even this analysis is delicate: the integral operators are strongly singular (in contrast to the 2D case), and well-posedness can only be established in L^2 spaces.

3. The development of numerical methods for integral equations on unbounded surfaces, in particular the boundary integral equations arising from rough surface scattering problems, stability and convergence analyses for these methods, analysis of effects of 'finite section' errors (truncating the domain of integration), and efficient matrix-vector multiply and iterative solution schemes. Key developments include:

(i) for the 2D case, discretisation and finite section error stability and convergence proofs and error estimates [J2,J3], together with analysis of an effective modified two-grid solver [J3];

(ii) an efficient matrix compression and matrix-vector multiply method, based on approximation of the discretised integral operator by sums of products of diagonal and Toeplitz matrices (block Toeplitz with Toeplitz blocks in the 3D case) [C5];

(iii) for a particular problem of scattering by an unbounded surface, a boundary element scheme, using basis functions which are products of piecewise polynomials and plane waves, which, with appropriate mesh grading can be shown to require a number of degrees of freedom increasing only logarithmically with the wavenumber [J11] to maintain accuracy - in [J13] the scheme is improved to obtain wavenumber independence. Applications to scattering by bounded obstacles are also sketched in [J11].

4. The development of practically useful algorithms and software, based on boundary integral equation formulations and the results of 1 and 2, for two- and three-dimensional boundary value problems for the Helmholtz equation, in particular problems of scattering by rough surfaces. Software, based on the 2D algorithms in [J3], coupled with a partial spatial Fourier transform to produce a $2\frac{1}{2}$D code, has been supplied under license to TRL Limited, to simulate propagation from a point source over a flat, piecewise constant impedance surface. Software has also been produced and supplied to TRL to model propagation over an uneven, varying impedance surface, to simulate effects of terrain on outdoor propagation.

5. The development and analysis of algorithms and software for problems of inverse scattering by bounded and unbounded surfaces. Under this heading we have studied:

(i) Fréchet differentiability of the solution to the 2D Dirichlet rough surface scattering problem with respect to the boundary shape [J6], establishing the differentiability and characterising the derivative as the solution to a boundary value problem. This appears to be the first domain derivative result for a case where the boundary is of infinite extent. In the same paper, we provide, using the Fréchet differentiability results, the first rigorous proof of the validity of small perturbation approximations which are in wide use in the engineering literature. Using these results we show that a plane wave incident on a small perturbation of a flat surface can give rise to an unbounded scattered field.

(ii) Newton-type methods for inverse scattering problems for bounded [J1,B1] and unbounded [C4] surfaces. In particular, we have proved superlinear local convergence of regularized Newton updates towards a regularized solution and the convergence of the regularized solution towards the true solution when the data error tends to zero, both under the condition that the true solution has an analytic boundary and the normal derivative of the total field has no zeros on the boundary.

(iii) Point source methods (in the sense of [7]) for the inverse scattering problem, which enable the scattered field to be reconstructed up to the boundary from limited measurements. Once the wave field is reconstructed the location of the boundary can be determined as the locus where the boundary condition is satisfied. The method is reviewed and analysed for bounded scatterers in [B1]. A version of the point source method for inverse scattering by unbounded surfaces is proposed and analysed in [C3,C6,C8,C11,J14].

4. Research Impact and Benefits to Society

We expect our research to have very considerable impact, in a variety of ways, from stimulating further research internationally, to the direct use of our novel algorithms and software. For reasons of space we mention only a few illustrative examples:

- [J11] will give hope that efficient boundary element schemes can be developed to work at high frequency using plane wave basis functions, and that their performance can be effectively tuned and understood by careful numerical analysis. As illustrated by the other articles in the issue of Phil. Trans. of which [J11] is part, the development of effective high frequency algorithms is an exciting and developing area.
- We envisage that the matrix compression and matrix-vector multiply scheme we have described in [C5] will, when combined with the multilevel approach recently proposed in [6], prove to be the method of choice in engineering practice for the accurate and efficient simulation of rough surface scattering problems.
- We envisage a very fruitful combination of the ideas of [J4,J5,J8] with other approaches to the study of operator equations on unbounded domains (e.g. the articles by Lindner & Silbermann and by Rabinovich & Roch in [5]) which do not use the (rather weak) notions of compactness which we exploit. Indeed an application for an EU Fellowship to bring Lindner from Chemnitz to Reading to pursue this synthesis has just been made.
- The monograph [B1] has been very positively reviewed in *Mathematical Reviews* and in *SIAM Review* and is expected to contribute internationally to the education of graduate students and to the stimulation of further research in inverse scattering.
- We expect that our work on the *inverse surface scattering* problem [C3,C6,C8,C11,J14] will stimulate further activity in this direction. Indeed, to encourage this along, we have, with Paul Martin, John DeSanto (Colorado School of Mines) and Alexander Voronovich (NOAA, Boulder, Colorado) just applied to run an AMS-IMS-SIAM Summer Research Conference on this theme.
- We note that the boundary element software we have supplied to TRL has already been used, in their own contracts with the Highways Agency, to produce predictions of the effect of land use and ground profile on road traffic noise propagation.

5. Dissemination

Considerable effort has been expended in effective dissemination of the research results. Mechanisms for dissemination have included:

- Publication of research results in book, refereed journal, and conference proceeding form (see the Appendix).
- Over 40 presentations at conferences by the PI, the Research Staff, and the associated PhD students (Anja Meier and Claire Lines), targeting acoustics, electromagnetics, waves, inverse problems, applied mathematics, and numerical analysis meetings. This includes **plenary invited lectures** at the INRIA/SIAM *5th International Conference on Mathematical and Numerical Aspects of Wave Propagation Phenomena*, Santiago de Compostela, Spain, July 2000, at *JEE '02, the European Symposium in Numerical Methods in Electromagnetics*, Toulouse,

March 2002, and at the *Mathematics of Finite Elements and Applications*, Brunel University, June 2003. Invited papers on results of the research were also given at: the LMS Durham Symposium on *Computational Methods for Wave Propagation in Direct Scattering*, July 2002, at an NSF-CBMS Conference on *Numerical Methods in Forward and Inverse Electromagnetic Scattering*, Colorado School of Mines, June 2002, at an Isaac Newton Institute Workshop on *Mathematical Challenges in Scientific and Engineering Computation*, at a UNESCO/Georgian Academy of Sciences conference on *Potential Theory: Applications in Solid Mecahnics and Acoustic and Electromagnetic Scattering*, Tbilisi, October 2003, at three Mathematisches Forschungsinstitut Oberwolfach workshops (on Boundary Element Methods, Inverse Problems, and Algorithms for the Arithmetic of Dense Matrices), and at numerous one day meetings.

- Over 20 invited research seminars given by the PI and the research staff.
- Organising a minisymposium on Direct and Inverse Surface Scattering at the JEE '02 Electromagnetics Symposium in Toulouse, March 2002.
- Licensing software (Noiseprop) to the Transport Research Laboratory Limited to simulate outdoor noise propagation over rough ground surfaces
- Organising the *IMA 3rd International Conference on Boundary Integral Methods: Theory and Applications* (Reading, September 2004), and seeking (successfully) EPSRC funding for a *Workshop on Developments in Boundary Element Methods for Acoustics and Electromagnetics* as part of the larger conference . Many of the final results of the project will be presented at this meeting.

6. Explanation of Expenditure

There was no significant variance from the original spending plans. The use of the budget for the major item (staff) has been discussed in section 2 above. EPSRC sponsored or organised events attended during the period of the grant include: an EPSRC-funded *Workshop on Boundary Element Techniques in Computational Acoustics and Electromagnetics*, September 2000, University of Bath, an EPSRC-funded LMS Durham Symposium on *Computational Methods for Wave Propagation in Direct Scattering*. Posters and/or papers were presented at three EPSRC-organised one-day meetings: an Acoustics Theme Day, a meeting on the CPDEs/Computational Engineering Mathematics Initiative, and an EPSRC-organised day at the Isaac Newton Institute, January 2003.

Appendix: List of Project Outputs

Publications listed are those in print, accepted, or submitted arising wholly or in part from work on this project. A number of other publications are still in preparation. Web links are given for the full text of the submitted articles. Preprints of many of the other articles are available at `www.personal.rdg.ac.uk/~sms03snc`

Books

[B1] R Potthast 2001 *Point Sources and Multipoles in Inverse Scattering Theory.* CRC/Chapman & Hall.

Refereed Journals

[J1] R Potthast 2001 *Inverse Problems,* **17,** 1419-1434. On the convergence of Newton's method in inverse scattering.

[J2] A Meier and S N Chandler-Wilde 2001 *Mathematical Methods in the Applied Sciences,* **24,** 209-232. On the stability and convergence of the finite section method for integral equation formulations of rough surface scattering.

[J3] S N Chandler-Wilde, M Rahman, and C R Ross 2002 *Numerische Mathematik,* **93,** 1-51. A fast two-grid and finite section method for a class of integral equations on the real line with application to an acoustic scattering problem in the half-plane.

[J4] S N Chandler-Wilde and B Zhang 2002 *Journal of Integral Equations and Applications,* **14,** 11-52. A generalized collectively compact operator theory with an application to second kind integral equations on unbounded domains.

[J5] T Arens, S N Chandler-Wilde, and K O Haseloh 2002 *Journal of Mathematical Analysis and Applications,* **272,** 276-302. Solvability and spectral properties of integral equations on the real line: I. weighted spaces of continuous functions.

[J6] S N Chandler-Wilde and R Potthast 2002 *Proceedings of the Royal Society of London, Series A,* **458,** 2967-3001. The domain derivative in rough surface scattering and rigorous estimates for first order perturbation theory.

[J7] B Zhang and S N Chandler-Wilde 2003 *Mathematical Methods in the Applied Sciences,* **26,** 463-488. Integral equation methods for scattering by infinite rough surfaces.

[J8] T Arens, S N Chandler-Wilde, and K O Haseloh 2003 *Journal of Integral Equations and Applications,* **15,** 1-35. Solvability and spectral properties of integral equations on the real line: II. L^p spaces and applications.

[J9] D Natroshvili and S N Chandler-Wilde 2003 *Memoirs on Differential Equations and Mathematical Physics,* **30,** 51-103. The Dirichlet metaharmonic Green's function for unbounded regions.

[J10] D Natroshvili, T Arens, and S N Chandler-Wilde 2003 *Memoirs on Differential Equations and Mathematical Physics,* **30,** 105-146. Uniqueness, existence, and integral equation formulations for interface scattering problems.

[J11] S N Chandler-Wilde, S W Langdon, and L Ritter 2004 *Philosophical Transactions of the Royal Society, Series A.* (Published electronically January 2004, DOI: 10.1098/rsta.2003.1339) A high-wavenumber boundary-element method for an acoustic scattering problem.

[J12] S N Chandler-Wilde and K O Haseloh submitted to *Journal of Integral Equations and Operator Theory.* Solvability and Fredholm properties of integral equations on the half-line in weighted spaces. (`www.personal.rdg.ac.uk/~sms03snc/weightieot.pdf`)

[J13] S W Langdon and S N Chandler-Wilde submitted to *SIAM Journal on Numerical Analysis.* A wavenumber independent boundary element method for an acoustic scattering problem. (`www.newton.cam.ac.uk/preprints/NI03049.pdf`)

[J14] C D Lines and S N Chandler-Wilde submitted to *Computing.* A time domain point source method for inverse scattering by rough surfaces. (`www.personal.rdg.ac.uk/~sms03snc/specialcdlscw.pdf`)

Conference Proceedings

[C1] T Arens, S N Chandler-Wilde, and A Meier 2000 in *Proceedings of the 5th International Conference on Mathematical and Numerical Aspects of Wave Propagation,* (A Bermudez, D Gomez, C Hazard, P Joly, J E Roberts, eds.) Santiago de Compostela, Spain, July 2000, pp. 3-13. Philadelphia: SIAM. Integral equation methods for scattering by one-dimensional rough surfaces. **(Invited plenary paper.)**

[C2] A Meier and S N Chandler-Wilde 2000 in *Proceedings of the 5th International Conference on Mathematical and Numerical Aspects of Wave Propagation,* (A Bermudez, D Gomez, C Hazard, P Joly, J E Roberts, eds.) Santiago de Compostela, Spain, July 2000, pp. 3-13. Philadelphia: SIAM. On the stability and convergence of the finite section method for integral equation formulations of rough surface scattering.

[C3] C D Lines and S N Chandler-Wilde 2001 in *Proceedings of the 3rd UK Conference on Boundary Integral Methods,* (P.J. Harris, ed.) Brighton, September 2001, pp. 258-267. Brighton: University of Brighton Press. An integral equation method for inverse scattering by rough surfaces.

[C4] R Potthast 2002 in *Proceedings of JEE '02, the European Symposium in Numerical Methods in Electromagnetics,* Toulouse, March 2002. A Newton method for inverse scattering by rough surfaces.

[C5] S N Chandler-Wilde, B Birbiad, and A Meier 2002 in *Proceedings of JEE '02, the European Symposium in Numerical Methods in Electromagnetics,* Toulouse, March 2002. A BMIA canonical grid method for scattering by rough surfaces.

[C6] C D Lines and S N Chandler-Wilde 2002 in *Proceedings of JEE '02, the European Symposium in Numerical Methods in Electromagnetics,* Toulouse, March 2002. A point source method for inverse scattering by rough surfaces.

[C7] S N Chandler-Wilde, S Langdon, and L Ritter *Proceedings of the 6th International Conference on Mathematical and Numerical Aspects of Wave Propagation Phenomena,* (G C Cohen, E Heikkola, P Joly, eds.) University of Jyväskylä, Finland, July 2003, pp. 257-262. Berlin: Springer. A Galerkin boundary element method for a high frequency scattering problem.

[C8] C D Lines and S N Chandler-Wilde in *Proceedings of the 6th International Conference on Mathematical and Numerical Aspects of Wave Propagation Phenomena,* (G C Cohen, E Heikkola, P Joly, eds.) University of Jyväskylä, Finland, July 2003, pp. 637-642. Berlin: Springer. A time domain point source method for inverse scattering by rough surfaces.

[C9] S W Langdon and S N Chandler-Wilde 2003 in *Proceedings of the 4th UK Conference on Boundary Integral Methods,* (S. Amini, ed.), pp. 67-76. Salford University Press. A Galerkin boundary element method for an acoustic scattering problem with convergence rate independent of frequency.

[C10] S W Langdon and S N Chandler-Wilde 2003 in *Proceedings of the Institute of Acoustics,* **25**(5), 224-233. A GTD-based boundary element method for a surface scattering problem.

[C11] C D Lines and S N Chandler-Wilde 2003 in *Proceedings of the Institute of Acoustics,* **25**(5), 234-242. Inverse scattering and field extrapolation for rough surface scattering.

References

[1] *PIERS 2002 Proceedings.* Cambridge, MA, The Electromagnetic Academy.

[2] *Acoustic Characteristics of Surfaces: Measurement, Prediction, and Applications. Proceedings of the Institute of Acoustics, Vol. 25, Part 5.*, 2002.

[3] R. C. Buck. Bounded continuous functions on a locally compact space. *Michigan Math. J.*, 5:95–104, 1958.

[4] J. A. DeSanto. Scattering by rough surfaces. In R. Pike and P. Sabatier, editors, *Scattering: Scattering and Inverse Scattering in Pure and Applied Science*, pages 15–36. Academic Press, 2002.

[5] I. Gohberg, A. Ferreira dos Santos, F.-O. Speck, F. Sepulveda Teixeira, and W.L. Wendland, editors. *Operator Theoretical Methods and Applications to Mathematical Physics: The Erhard Meister Memorial Volume*. Birkhäuser, 2004.

[6] S.-Q. Li, C. H. Chan, M.-Y. Xia, B. Zhang, and L. Tsang. Multilevel expansion of the sparse-matrix canonical grid method for two-dimensional random rough surfaces. *IEEE Trans. Antennas Propagat.*, 49:1579–1589, 2001.

[7] R. Potthast. A point source method for inverse acoustic and electromagnetic obstacle scattering problems. *IMA J. Appl. Math.*, 61:119–140, 1998.

[8] M. Saillard and A. Sentenac. Rigorous solutions for electromagnetic scattering from rough surfaces. *Waves Random Media*, 11:R103–R137, 2001.

[9] A. G. Voronovich. *Wave Scattering from Rough Surfaces.* Springer, Berlin, 2nd edition, 1999.

[10] K.F. Warnick and W. C. Chew. Numerical simulation methods for rough surface scattering. *Waves Random Media*, 11:R1–R30, 2001.