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# Electron temperature measurements by incoherent scattering in the presence of strong small scale temperature irregularities

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#### Abstract

The results of electron temperature measurements by incoherent scattering have been considered under the conditions of existence of strong small scale temperature irregularities in the F-region of the ionosphere. A method to determine such local temperature enhancement from the analysis of the mean square error in the ACF fitting and the total scattered power of the ISR signal is pointed out. © 1998 Published by Elsevier Science B.V.

# 1. Introduction

One of the most significant new physical phenomena, discovered during ionospheric modification by powerful radio waves, is the generation of small scale striations, plasma density depletions strongly elongated along the Earth's magnetic field [1,2]. Recently such striations were also observed in experiments in situ on board of rockets [3,4]. They have been seen as essentially local stationary depletions of the plasma density  $|\delta N/N| \sim 0.05$  with scales of the order of 5 meters across and several kilometers along the magnetic field lines.

A nonlinear theory determining the conditions of existence and structure of stationary striations was recently developed [5]. The theory is in remarkably good agreement with the rocket observations [3]. Its most significant prediction is a strong local enhancement of the electron temperature  $T_e$  in the striations. The maximal electron temperature in the center of stri-

ations can reach values  $T_e/T_i \sim 3-5$ , average temperature ratios in a strongly disturbed region can be of the order  $T_e/T_i \sim 2-3$ .

On the other hand, the measurements by incoherent scattering of the electron heating in ionospheric modification experiments often show a modest level of temperature ratios,  $T_e/T_i \leq 1.2-1.3$  [6,7]. It is important to remember, however, that in the analysis of the incoherent scattering measurements  $T_e$  is assumed to be constant in the scattering volume. The radial dimension of this volume is 600 m in Arecibo and 5 km in Tromsø, the lengths 150-600 m. But according to the theory [5], in the ionosphere, modified by radiowaves, the electron temperature can be strongly inhomogeneous, due to the existence of striations, with  $T_e/T_i$  changes of 3-5 to 1 on the scale of 5 m.

The goal of this Letter is to consider the effective shape of the ion line spectra or its Fourier transform, the autocorrelation function (ACF), when there are strong local electron temperature variations. In particular, we will check whether the modest increase in the electron temperature observed by incoherent scattering is compatible with the large local temperature variations in an inhomogeneously heated ionosphere. We will also check the possibility to detect the effect of strong local heating.

### 2. Results

Here we restrict our consideration to the case of an unmagnetized plasma. In typical ionospheric conditions  $k\lambda_D \gg 1$ , where k is a wave vector and  $\lambda_D$  is the Debye length. The scattering cross section in this limit according to Refs. [8,9] has the form

$$\sigma_{\omega} = \frac{N\sigma_{\rm e}/\pi^{1/2}\Omega_{\rm i}}{[1+z_{\rm i}RT_{\rm e}/(r)T_{\rm i}]^2 + z_{\rm i}^2I^2T_{\rm e}^2(r)/T_{\rm i}^2} \times \exp\left(-\frac{\omega^2}{\Omega_{\rm i}^2}\right)$$
(1)

Here  $T_{e,i}$  is the temperature of electron and ions, N is the electron density,  $\sigma_e$  is the Thomson scattering cross section,  $\Omega_i = k(2T_i/M_i)^{1/2}$  is the ion frequency, and  $z_i$  is the ion charge number. Functions R and I are defined by the following relations,

$$R = 1 - 2\frac{\omega}{\Omega_{\rm i}} \exp\left(-\frac{\omega^2}{\Omega_{\rm i}^2}\right) \int_{0}^{\omega/\Omega_{\rm i}} {\rm e}^{y^2} {\rm d}y,$$
$$I = \sqrt{\pi} \frac{\omega}{\Omega_{\rm i}} \exp\left(-\frac{\omega^2}{\Omega_{\rm i}^2}\right).$$
(2)

The incoherent nature of the scattering from the different elementary parts within the scattering volume allows us to regard the scattering as a superposition of independent scatterers, each with its own spectrum. The observed signal is therefore determined by averaging over the contributions from the whole scattering volume. To determine the cross section we only need to integrate expression (1) over the scattering volume V,

$$\langle \sigma_{\omega} \rangle = \frac{1}{V} \int \sigma_{\omega} \, \mathrm{d}^3 \boldsymbol{r} \,.$$
 (3)

When we consider the ionosphere as modified by a powerful HF radiowave, we may neglect the density fluctuations in the averaging (3) since according to both theory and observations [3,5] they are of the order of only a few percent.

To stress the effect of horizontal temperature inhomogeneity we consider the most perturbed situation when striations are packed closely together as shown in Fig. 1. The results of our calculations are shown in Fig. 2 where the spectral form of the ion plasma lines in the disturbed ionosphere is presented for different values of the maximum electron temperature in the center of striations  $T_{\rm m} = T_{\rm e}^{\rm max}/T_{\rm i}$ . The value of  $T_{\rm m}$  as is shown in Ref. [5] increases with the power of the heating wave. For comparison, the ion plasma lines for a homogeneously heated plasma are shown in Fig. 3 for different values of  $T_{\rm e}/T_{\rm i}$ . We see that their form is quite different from the inhomogeneous case (Fig. 2). To match our results with the usual theory we have shown in Fig. 4 the form of the ion line for a homogeneously heated plasma, which is the best least-squares fit for the ion line obtained in disturbed conditions. The electron temperature determined from this fit,  $T_e/T_i = 1.4$ , is quite different from both the maximum,  $T_{\rm m} = 6$ , and the average,  $T_{\rm e}/T_{\rm i} = 1.9$ , electron temperature in striations. So, we see that the ion plasma line does not give real information about the electron temperature in a locally heated plasma. It is easy to understand the physical reason for this effect considering the classical ion plasma lines in uniform conditions, presented in Fig. 3. One can see that the input from the strongly heated plasma  $T_e/T_i \gg 1$  is much less than from the cold one. It means that in averaging the cold part dominates.

In Fig. 5 is shown the autocorrelation function which is proportional to the cos-Fourier transformation of  $\sigma_{\omega}$  for the inhomogeneously heated ionospheric plasma. The height dependence of its basic parameters  $\tau_0$  and  $\tau_1$ , characterizing the first zero and the first minimum points, are shown in Table 1.

In practice, the ionospheric parameter estimation starts from a multiparameter fit of the ACF for a model of a homogeneous plasma. We produced the one parameter least-squares fit of the ACF in disturbed conditions, like that depicted in Fig. 1, for a model of a homogeneously heated plasma. The calculation of the ACF for the disturbed conditions has been done at different values of the maximal temperature  $T_{m}$ . The parameter which has to be determined is as before the ratio of the electron temperature to the ion temperature,  $T_{e}/T_{i}$ . The other parameters such as the ion temTable 1

Parameters of the ionosphere (daytime) and the dependences of the basic ACF parameters, the first zero and the first minimum points ( $\tau_0$  and  $\tau_i$ , see Fig. 5) on the height at a given maximal electron temperature in striations  $T_m$ . The value of  $\tau_0$  and  $\tau_i$  are normalized to the characteristic time  $\tau_c = 2 \times 10^{-4}$  s (224 MHz/f)( $M_{i0}/16$ )<sup>1/2</sup>(1000 K/ $T_{i0}$ )<sup>1/2</sup>,  $T_{i0}$  is the ion temperature, f is the frequency of a radar, and  $M_{i0}$  is the ion mass.

Height (km)	Density (cm <sup>-3</sup> )	Electron temperature (K)	Ion temperature (K)	$T_{\rm m}=2$		$T_{\rm m}=6$	
				$\overline{x_0}$	$x_1$	<i>x</i> <sub>0</sub>	<i>x</i> <sub>1</sub>
200	5.0E+05	1300	1100	1.6	2.4	1.5	2.2
250	1.0E+06	1700	1300	1.4	2.4	1.3	2
300	1.6E+06	2000	1400	1.3	2	1.2	1.8
400	1.5E+06	2400	1450	1.2	1.9	1.1	1.7



Fig. 1. A set of striations (temperature distribution  $T = T_c/T_i$  as a function of coordinate,  $X_c \sim 1$  m is a characteristic striation scale).

perature  $T_i$ , the electron density and the O<sup>+</sup> content are assumed to be known. The results are presented in Table 2 where the maximal temperature  $T_m$  is shown against the result of the best fitting. It is seen in accordance with Fig. 4 that the determined value of  $T_e/T_i$ is significantly underestimated.

The fourth column of the table contains the mean square deviation of the fitted ACF  $f_{acf}$  and the model  $f_{acm}$ ,  $\Delta_f = (1/j) \sum_j (f_{acm}^j - f_{acf}^j)^2 / f_{acm}^{j2}$ , j is the number of data points. The analysis of the error in the fitting procedure may provide the most important information about ionospheric plasma state. In Refs. [10,11], such an analysis was performed for the errors resulting from small variations of ionospheric parameters, but no regular structure has been assumed. In



Fig. 2. The spectral form of the ion plasma line  $\sigma_{\omega} \Omega_i / N$  for a homogeneously heated plasma at different values of  $T_e / T_i$ , z = 250 km.

our case the strong local heating may be also treated as a variation of plasma parameters. That is expected to lead to a substantial increase of the error obtained. We consider the deviation of the best fit and model ACF calculated for a disturbed ionosphere. The dependence of the error obtained by fitting against the maximal temperature  $T_m$  is shown in Fig. 6. The deviation has been normalized to the standard error of the fitting  $\Delta_r = 1\%$  [10]. We see, that at  $T_m > 3$  the error induced by the regular inhomogeneous structure of  $T_e$ is much higher than the standard error and it rapidly increases with  $T_m$ .

The other possibility to detect strong local heating appearance analysing the dependence of the total power of the scattered signal W on the temperature

Table 2 The results of the ACF fitting

Maximal electron temperature	Fitted value of electron temperature	Average electron temperature	Error of the fitting $\Delta_f/\Delta_r$
2	1.14	1.2	0.295
4	1.32	1.5	5.977
6	1.43	1.9	14.229
8	1.50	2.2	22.503

0.60



Fig. 3. The spectral form of the ion plasma line  $\sigma_{\omega}\Omega_i/N$  for an inhomogeneously heated plasma at various values of  $T_{\rm m}$ , z = 250 km.



Fig. 5. The autocorrelation function for an inhomogeneously heated plasma at height z = 200 km and  $T_m = 2$ .



Fig. 4. The spectral form of the ion plasma line  $\sigma_{\omega} \Omega_i / N$  for an inhomogeneously heated plasma and the best fit for a homogeneous plasma, z = 250 km.

Fig. 6. The dependence of the relative error of the fitting  $\Delta_f/\Delta_r$  on the maximal electron temperature  $T_m$ ,  $\Delta_r = 1\%$ , the insert shows the relative error dependence on the fitting value of  $T_e/T_i$ .

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Fig. 7. The dependence of the function  $g(T_m)/g(2) = W_0/W - 1$ on the maximal electron temperature  $T_m$  for a homogeneously and an inhomogeneously heated plasma at various heights.

variation. The usual dependence in inhomogeneous media is  $W = W_0/(1 + T_e/T_i)$ . We plot the function  $g = W_0/W - 1$  in Fig. 7 for the frequency integrated power spectrum (1) in an inhomogeneously heated plasma. In a homogeneous plasma g is a linear function of  $T_e/T_i$ , and this is shown in Fig. 7 by the solid and dotted straight lines. The real dependence in inhomogeneous conditions shown in the figure for different ionospheric heights is far from linear.

Let us emphasize that according to Refs. [2,5] localized heating appears in striations when there is strong anomalous absorption of radiowaves. Such anomalous absorption has been observed regularly at the SURA and Tromsø facilities. According to Tromsø, the ISR data electron temperature does not grow significantly in spite of strong anomalous absorption and the high power of the HF heating radio waves, for example, under  $P \ge 220$  MW,  $T_e/T_i \le 1.2$ , see Ref. [7]. From the present analysis, it is clear that the strong heating in the striations cannot be observed directly because of the averaging over the small scale striations. The modest temperature increase observed in practice is, therefore fully compatible with a large local temperature increase in the striations predicted by the theory.

Note that for conditions of weak energy loss rate by thermal electrons (such conditions could be realized at F-region altitudes at nighttime due to lower electron concentrations in a low latitude ionosphere), the main bulk of electrons could be significantly heated [13]. Events of this type were possibly observed in Arecibo winter nighttime heating experiments [14–16].

## 3. Conclusion

The theory predicts a strong enhancement of the electron temperature inside small scale striations. This enhancement is not seen in ISB observations. In this paper we have considered in detail the incoherent scattering process in the presence of strongly heated small scale irregularities in the ionosphere and showed that there is no contradiction between the existing ISR observational data and the theory.

Based on the present analysis we can propose the following method to determine the existence of strong local enhancement of electron temperature inside striations using an incoherent scattering radar. In the nondisturbed ionosphere the electron and ion temperature, and the ISR standard error of mean square fitting of the ACF  $\Delta_r$  should be determined. Then, the same procedure should be performed after the powerful heater is turned on. If strong heating in striations exists, the mean square fit error of the ACF should grow significantly. From comparison of this error with the curve given in Fig. 6 one can determine an effective value of  $T_{\rm m}$ . The error should increase with the heater power according to Ref. [5] and Fig. 6. The dependence of the total power of the scattered ISR signal could be compared with the one presented also in Fig. 7.

Note that one can expect that analogous small scale strong plasma temperature and density variations could arise in the natural auroral ionosphere in strongly disturbed conditions. The proposed method could be used for the analysis of the state of ionospheric plasma in this case also.

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